Slicing a Block into Pieces: A Novel Tree Structure to Capture Sequential Exercise Policy

Liu, Liang-Chih [*] Dai, Tian-Shyr [†] Chang, Hao-Han [‡]

January 29, 2021

Abstract

This paper presents a novel state-transition tree to evaluate corporate securities when the granted options for the security holders are exercised bit by bit. The tree is developed to accurately capture the whole paths of sequential exercises resulting from share dilution and (or) capital injection that change the values of underlying assets when options are exercised. It thus shows significant valuation impact when the pricing results are compared to the ones from the traditional models with the block exercise constraint. This structure can then be well applied not only to a risk-neutral valuation of tradeable convertible bonds but also to a utility-based valuation of nontransferable employee stock options for which close-form solutions are unavailable.

Keywords: block exercise, sequential exercise, convertible bonds, employee stock options

[*]Department of Information and Finance Management, National Taipei University of Technology, No.1, Sec.3, Zhongxiao E. Rd., Daan Dist. Taipei City, Taiwan. E-mail: lcliu@ntut.edu.tw.

[†]Department of Information Management and Finance, Institute of Finance, National Chiao-Tung University, No.1001 Daxue Rd, Hsinchu City, Taiwan. E-mail: cameldai@mail.nctu.edu.tw.

[‡]Institute of Finance, National Chiao-Tung University, No.1001 Daxue Rd., Hsinchu City, Taiwan. E-mail: sonicstars@gmail.com.

1 Introduction

Theoretical valuation of corporate securities relies heavily on the assumption that security holders will pursue the best of their interest by choosing the timing to wholly exercise their granted options as a block. However, the appropriateness of such block constraint has sparked off an intense debate since the early 1980s (see Emanuel, 1983; Constantinides and Grundy, 1987; Spatt and Sterbenz, 1988). One of the arguments centered on how the exercises of options change the issuing firms' capital structure and thus the values of the underlying assets to further shift the original block exercise policy toward new one. Indeed, empirical evidence does not always lend support to block exercises. Some security holders prefer to distribute the exercises of their options over several dates (see Hemmer et al., 1994; Bühler and Koziol, 2002), but up to now it is still a big challenge to incorporate such sequential exercise policy into valuation models.

Convertible bonds (abbreviated as `CB`s) are one of the notable examples. Though `CB` holders can share the issuers' upside potential profits with equity holders by exercising their conversion options, the values of the newly converted stocks will decrease with the increment of conversion volumes due to share dilution effect. Therefore, there exists an optimal conversion policy for the `CB` holders to trade their benefits off against the dilution effect. By spreading out the conversions over time to minimize the dilution effect, the sum of the conversion payoffs and thus the `CB` values are maximized. Sequential conversion could be an adoptable policy as observed in Bühler and Koziol (2002) and should be considered when a standard risk-neutral valuation of `CB`s is applied.

Employee or executive stock options (abbreviated as `ESO`s) are another well-known examples. In addition to capital injection, the exercises of `ESO`s also bring share dilution. However, unlike `CB`s that are tradeable assets, a risk-neutral valuation of nontransferable `ESO`s is inappropriate, because employees cannot hedge their `ESO`s by short selling the underlying stocks. Therefore, rather than treating employees as risk-neutral option holders, a utility-based valuation of `ESO`s is applied to associate the risk preferences of the employees with their exercise policies. Along with the capital injection and share dilution effect, risk-aversion provides another source of incentives for employees to early exercise their options bit by bit as demonstrated by Hemmer et al. (1994); Aboody (1996); Huddart and Lang (1996).

Some models indeed accommodate the possibility of intertemporal exercises, but we found that they all tackle this issue in very limited cases. Constantinides and Grundy (1987) evaluates `CB`s in ignorance of their call and credit risk. However, the presence of call provisions may create salient evaluation impact on `CB`s, for the conversion policy is likely to be influenced by the `CB` issuers' (intended) call decision (see Chen et al., 2013; Bechmann et al., 2014). Furthermore, the call decision may also interact with the issuers' default decision (see Acharya and Carpenter, 2002; Kim and Stock, 2014). Bühler and Koziol (2002) evaluate otherwise identical `CB`s considering credit risk, but their model does not allow premature default. When `ESO`s are evaluated, Emanuel (1983) and Spatt and Sterbenz (1988) consider unlevered issuers in ignorance of the risk preferences of their employees. While a utility-based valuation is applied, Jain and Subramanian (2004) and Rogers and Scheinkman (2007) treat the underlying stocks as exogenous variables failing to capture how capital injection and share dilution influence the stock prices when `ESO`s are exercised.

In this paper, a novel quantitative framework, the "state-transition tree", is developed to systematically incorporate sequential exercises into a structural credit risk model viewing corporate securities as contingent claims on their issuing firms' assets (see Merton, 1974). Though the tree method discretely specifies the asset values, its flexibility is exploited to accommodate different capital structure, payout policies, and default boundaries for levered firms as in Wang et al. (2014). Besides, by treating stock prices as endogenous variables associated with asset values and the amounts of outstanding shares, the impacts of capital injection and share dilution on the prices can be faithfully reflected. In particular, each asset value node in a state-transition tree can especially capture several possible states of cumulative exercise amounts to thus reveal different amounts of outstanding shares for a given asset value. Intertemporal exercises are then traced via a transition between two intertemporal asset value nodes. To capture capital injection that raises asset values, more than one state-transition tree is combined in layers to be a forest as in Liu et al. (2016); each tree captures one possible scenario of capital structure, for instance before and after capital injection. Via the backward induction in a state-transition tree or a forest, the entire paths of intertemporal exercises are captured; this procedure is adoptable for not only a risk-neutral valuation of tradeable CBs but also a utility-based valuation of nontransferable ESOs. To the best of our knowledge, our proposed framework is the most comprehensive one being able to accommodate sequential exercise policy.

We examine the valuation impact of allowing for sequential conversion policy on CBs. Compared with the CB values implied by block conversion policy, accommodating sequential conversions creates greater valuation impact on noncallable CBs than the callable counterparts, because the presence of call provisions could force CB holders to cluster their conversion at suboptimal timings for avoidance of (impending) calls that curtail conversion values. This impact persists for different choices of conversion ratios and is getting salient when the interest rate level decreases or when both of the asset value volatility and cash dividends increase. That makes our framework increasingly important when CBs are evaluated, because empirical evidence shows that CBs are more likely to be issued by unrated or speculative firms (see Huang and Ramirez, 2010), whose stock prices tend to be volatile. In addition, firms are more likely to issue noncallable CBs to cater to the preferences of convertible arbitrage hedge funds, the dominant investors in the CB market since 2005 (see De Jong et al., 2013; Grundy and Verwijmeren, 2018). That is because the presence of call risk exposes these investors to the risk of unsuccessful conversion arbitrages, and such risk is especially significant when the interest rate is in a downtrend.

We then examine the valuation impact of allowing for sequential exercise policy on ESOs. Consistent with Jain and Subramanian (2004), the ESO values implied by block exercise policy are underestimated when the employees' relative risk aversion or asset value volatility exceeds a certain threshold, if they are compared with the ESO values accommodating sequential exercises. However, in contrast with Jain and Subramanian (2004), our framework captures capital injection and share dilution due to exercises of ESOs to make the total ESO values increase with the outstanding amount of the ESOs in a relatively concave way, because the dilution effect is significant enough to suppress the underlying stock prices and thus the ESO values. That makes our framework especially contribute to the valuation when the issuing firms' capital structure contains substantial ESOs, such as the firms in high-technology industries (see Huddart and Lang, 1996).[1]

The remainder of this paper proceeds as follows. The structural credit risk models and the trees accommodating the models are introduced in Section 2. Section 3 describes how a state-transition tree and the forest are constructed to capture sequential exercise policy. Section 4 illustrates the valuation impact of allowing for sequential exercise policy on CBs and ESOs when a risk-neutral valuation and a utility-based valuation are applied, respectively. Section 5 concludes this paper.

[1]According to Huddart and Lang (1996)'s sample, the mean number of options granted by firms in high-technology industries per year could reach 32.4% of shares outstanding.

2 Basic terms and preliminaries

In this section, we first introduce the settings adopted by structural credit risk models. Then, we introduce the tree structure accommodating structural models with different settings.

2.1 Structural Credit Risk Models

2.1.1 Firm Value Dynamics and Capital Structure

A structural model specifies the dynamics of the market value of a firm's asset and the condition leading to bankruptcy filing. Under the model, all of the securities issued by the firm are viewed as the contingent claims on the firm's unlevered asset. Here we follow Merton (1974) by assuming that the firm's unlevered asset value at time t, V_t, obeys the following process under the risk-neutral probability measure $\mathbf{P}$:

$$dV_t = rV_t dt - Y_t + \sigma V_t dz. \tag{1}$$

We follow Attaoui and Poncet (2013) by setting r to the long-term average interest rate, because the impact of stochastic interest rate can be negligible, as suggested in Ju and Ou-Yang (2006). Y_t indicates the payout for dividends or contractually-obligated payments at time t. Its form depends on different settings of payout policies and will be discussed in Section 2.1.2. It can be observed from **Equation (1)** that the firm's asset value follows a lognormal diffusion process between two payouts. σ denotes the volatility and is regarded as a proxy for the firm's business risk. We follow Fan and Sundaresan (2000) by setting σ to a constant, since the firm manager cannot alter the business risk arbitrarily at any time because of restrictive covenants in the outstanding bonds of the firm. dz denotes a standard Brownian motion. If the financial market is complete, there exists a corresponding real world probability measure $\mathbf{Q}$ equivalent to $\mathbf{P}$ (see Harrison and Kreps, 1979). Under the measure $\mathbf{Q}$, the standard Brownian motion dw governs the randomness of V_t whose expected rate of return is η, $\eta \geq r$.

We consider that the firm has the capital structure comprising N shares of common stocks, a T-year straight bond (`SB`) with the face value F_s and coupon rate C_s, a security Λ that grants its representative holder the right to obtain the firm's newly-issued common stocks. The Λ could be the one comprised by N_c units of T-year `ESO`s or a T-year junior `CB` with the face value F_c and coupon rate C_c. If the `CB` is callable, the issuer can prematurely redeem it at the call price K_c after the call protection period, and the holder can convert it into λ shares of the newly-issued common stocks. Therefore, `CB` conversion dilutes the existing share holders' value. On the other hand, each `ESO` grants its holder the right to buy one share (i.e., $\lambda = 1$) of the newly-issued common stock at the price K_c after the vesting period. The exercise of `ESO`s not only dilutes the share holders' value but injects capital into the firm.

If all of these securities are contingent claims on the issuing firm's unlevered asset, we denote the value of the firm's equity (`E`) at time t by $E(t, V_t)$, straight bond by $SB(t, V_t)$, and security Λ by $\Lambda(t, V_t)$. The $\Lambda(t, V_t)$ could be $CB(t, V_t)$ or $ESO(t, V_t)$. The total levered firm's asset value at time $t = 0$ is thus expressed as

$$V_0^L = E(0, V_0) + SB(0, V_0) + \Lambda(0, V_0), \tag{2}$$

where $E(0, V_0) = N \cdot S(0, V_0)$. $S(0, V_0)$ is the initial stock price considering future share dilution and capital injection (if existent).

Two kinds of market frictions are considered. When using debt capital, the firm could announce default on its debt immediately prior to bankruptcy filing. As long as the firm is solvent, its coupon payments are tax-deductible at rate τ, where $\tau \in (0, 1)$. Once the firm is liquidated after bankruptcy, a constant fraction ω of the firm's unlevered asset value is lost as liquidation costs such as the legal fee (see Leland, 1994). Then the remaining asset value is distributed according to the absolute priority rule as reported empirically by Bris et al. (2006).

2.1.2 Payout Policy

The Y_t in **Equation (1)** denotes the firm's payout for dividends or debt repayments at time t, and its value relies on the setting of the firm's payout policy adopted by a structural model. There are two popular settings listed as follows.

- **(P.1)** The firm is assumed to continuously pay parts of its asset value V_t at a rate δ to service its dividend payments and debt repayments (i.e., $Y_t = \delta V_t dt$). If the payout $\delta V_t dt$ exceeds the debt repayments, the remaining part goes to share holders as dividend payments. Otherwise, the share holders need absorb the deficiencies (see Kim et al., 1993; Collin-Dufresne and Goldstein, 2001; Attaoui and Poncet, 2013).

- **(P.2)** The firm is assumed to service all of its payments C_t^O at time t via its asset value (i.e., $Y_t = C_t^O dH_t$, where the H_t is a step function that increases by one at each payment date) (see Merton, 1974; Lando, 2004; Wang et al., 2014).

Once the δ in **(P.1)** is 0, it becomes the setting in Geske (1977) that all of the firm's payments are absorbed by the share holders. While **(P.1)** has already specified a given dividend policy, **(P.2)** can accommodate all types of policies such as the dividend smoothing policy (see Leary and Michaely, 2011; Chen et al., 2012).

For ease of illustration hereafter, we assume that both of the T-year `SB` and `CB` in **Equation (2)** continuously pay out a stream of coupon flows to their holders if their issuing firm is solvent. Then for each time $t \in (0, T)$, we let t^- and t^+ denote the times immediately before and after a payout.

2.1.3 Default Condition

In addition to a payout policy, a structural model needs specify a default condition when evaluating the contingent claims in **Equation (2)**. There are two strands of settings specified in relevant literature as follows.

- **(D.1)** For every contractually-obligated payment occurred at time $t \in (0, T]$, the default is triggered once the levered firm's equity value before the payment time t is less than the repayment. That is, the firm files for bankruptcy once $E(t^-, V_{t^-}) = 0$ (see Geske, 1977; Leland, 1994; Chen, 2010; He and Xiong, 2012; Kuehn and Schmid, 2014).

- **(D.2)** The default is triggered once the firm's asset value is less than a given boundary. That is, the firm files for bankruptcy once $V_{t^-} \leq D_{t^-}$ for $t \in (0, T]$ (see Black and Cox, 1976; Longstaff and Tuckman, 1994; Fan and Sundaresan, 2000).

Structural models with **(D.1)** and **(D.2)** are viewed as the ones incorporating endogenous and exogenous default boundaries, respectively (see Leland, 2004). A model with **(D.1)** implies that the firm will choose to file for bankruptcy once it no longer has the ability to service its debt repayments even by external equity financing (see Chen, 2010; He and Xiong, 2012; Kuehn and Schmid, 2014). A model with **(D.2)** on the other hand supposes that bankruptcy can be forced via the bond covenants such as the safety covenants (see Black and Cox, 1976; Briys and De Varenne, 1997), the minimum net worth covenants (see Leland, 1994), the maintenance covenants (see Longstaff and Tuckman, 1994), and the minimum net cash flow covenants (see Fan and Sundaresan, 2000). Though a default condition can be determined endogenously or exogenously, we in this paper assume that default occurs prior to any Λ holder's exercise decision.

2.1.4 The Representative Λ Holder's Policy to Exercise Options

Given a payout policy and default condition, the security Λ is evaluated considering the representative holder's policy to maximize her expected utility of the payoffs from the exercises of her options. In particular, if the Λ is a tradable asset such as the `CB` and are held by the representative investor operating in a complete market, then the risk-neutral valuation of such asset is applied. That is, all investment positions can be well hedged in that market, so required returns on risky assets are all independent of risk preferences of investors. Therefore, the risk-neutral `CB` investor has linear utility of each conversion payoff; she can maximize her expected utility of these intertemporal payoffs by just maximizing the present value of the `CB` (see Carpenter et al., 2010). In response to such conversion policy, the `CB` issuer will choose the call policy to maximize its share holders' value.

If the Λ is nontransferable asset such as the `ESO`s, the risk-neutral valuation is inappropriate because the representative employee cannot hedge her `ESO`s by short selling the underlying stocks (see Rogers and Scheinkman, 2007). Therefore, the utility-based valuation of the `ESO`s is adopted to account for the risk preferences of the employee. We follow Jain and Subramanian (2004) to assume that the firm can hedge the `ESO`s according to the exercise strategy of the representative employee, but the employee can not. The employee has constant relative risk aversion (CRRA) utility of the exercise payoff m occurred at time t

$$U(t, m) = e^{-\beta t} m^{\gamma}, \tag{3}$$

where $\gamma \in (0, 1]$, and her utility regarding all of the exercise payoffs is time-additive. Notice that $1 - \gamma$ is the coefficient of relative risk aversion. If the employee is risk-averse, $\gamma < 1$ so that the coefficient is positive. The β is the rate the employee used to discount her expected utility of each future payoff from the exercise of her `ESO`s. Accordingly, from the perspective of the `ESO` issuing firm, the nontransferable `ESO`s are evaluated under the risk-neutral probability measure $\mathbf{P}$ considering that the employee aims to maximize her expected utility of these intertemporal exercise payoffs, TU, under the corresponding real world probability measure $\mathbf{Q}$.

2.2 Trees to Accommodate Structural Credit Risk Models

2.2.1 Trees for the Firm Value Dynamics

A tree is a numerical method to characterize the evolution of a stochastic process. It divides a certain time interval into n equal-length time steps and discretely specifies the value of the stochastic process at each time step. Contingent claims on the stochastic process can be priced via backward induction in a tree if the stochastic process is modeled via that tree. Pricing results will converge to the theoretical values as $n \to \infty$ (see Duffie, 1996).

Notice that the firm value dynamics between two payments obeys a lognormal diffusion process (see **Equation (1)**). A CRR tree (see Cox et al., 1979) is a binomial tree characterizing such diffusion process via four parameters, u, d, P_u and P_d. u and d parameterize the state of the firm's asset value, from the initial value V either up to Vu or down to Vd at the next time step. P_u and P_d parameterize the probability for up and down movement of the firm's asset value for each time step. Define the V-log-price of the firm's asset value $V^{'}$ as $\ln(V^{'}/V)$ and the log-distance between V and $V^{'}$ is $|\ \ln V - \ln V^{'}|$. If a time interval $[0, T]$ is separated into n equal-length time steps and $\Delta t = T/n$, the mean (denoted by μ) and variance (denoted by `Var`) of the V_t-log-price of the $V_{t+\Delta t}$ are $(r - \sigma^2/2)\Delta t$ and $\sigma^2 \Delta t$, respectively. By asymptotically matching μ and `Var` via a one-step binomial structure, we obtain the four parameters as follows: $u = e^{\sigma\sqrt{\Delta t}}$, $d = 1/u$, $P_u = \left(e^{r\Delta t} - d\right)/(u - d)$, and $P_d = 1 - P_u$. The dashed branches in **Figure 1(a)** illustrates a one-step CRR tree. The log-distance between any two vertically adjacent CRR nodes (e.g., nodes G and H) is $2\sigma\sqrt{\Delta t}$.

To involve downward jumps in firm's asset values due to the presence of cash payments, the trinomial structures proposed by Dai and Lyuu (2010) are incorporated to avoid the node proliferation problem. **Figure 1(a)** illustrates an example for the presence of downward jumps. The cash payments at time $t = T/2$ are denoted by C_1^O and C_2^O, and they lead to downward jumps from nodes G and H to nodes I and J, respectively. The solid trinomial branches are used to connect the nodes deviating from original CRR nodes to the CRR nodes at the next time step. Take node I for example and denote the firm's asset value on node I by $v(I)$. At its next time step, there must exist an unique node M whose $v(I)$-log-price $\hat{\mu}$ lies in the interval $[\mu - \sigma\sqrt{\Delta t}, \mu + \sigma\sqrt{\Delta t})$. We then select node M and its adjacent nodes L and N to construct the trinomial structure from node I. Same procedure is also applied to node J. The corresponding branching probabilities p_u, p_m and p_d are determined by asymptotically matching μ and `Var`. If we define $f(t, V)$ as the discounted expected value of a contingent claim at time t when the firm's asset value is V, then its value on node I can be expressed in the form of backward induction on the trinomial structure:

$$f\left(t^+, V_{t^+}\right) \equiv e^{-r\Delta t}\left(p_u f((t+\Delta t)^-, V^u_{(t+\Delta t)^-}) + p_m f((t+\Delta t)^-, V^m_{(t+\Delta t)^-}) + p_d f((t+\Delta t)^-, V^d_{(t+\Delta t)^-})\right), \quad (4)$$

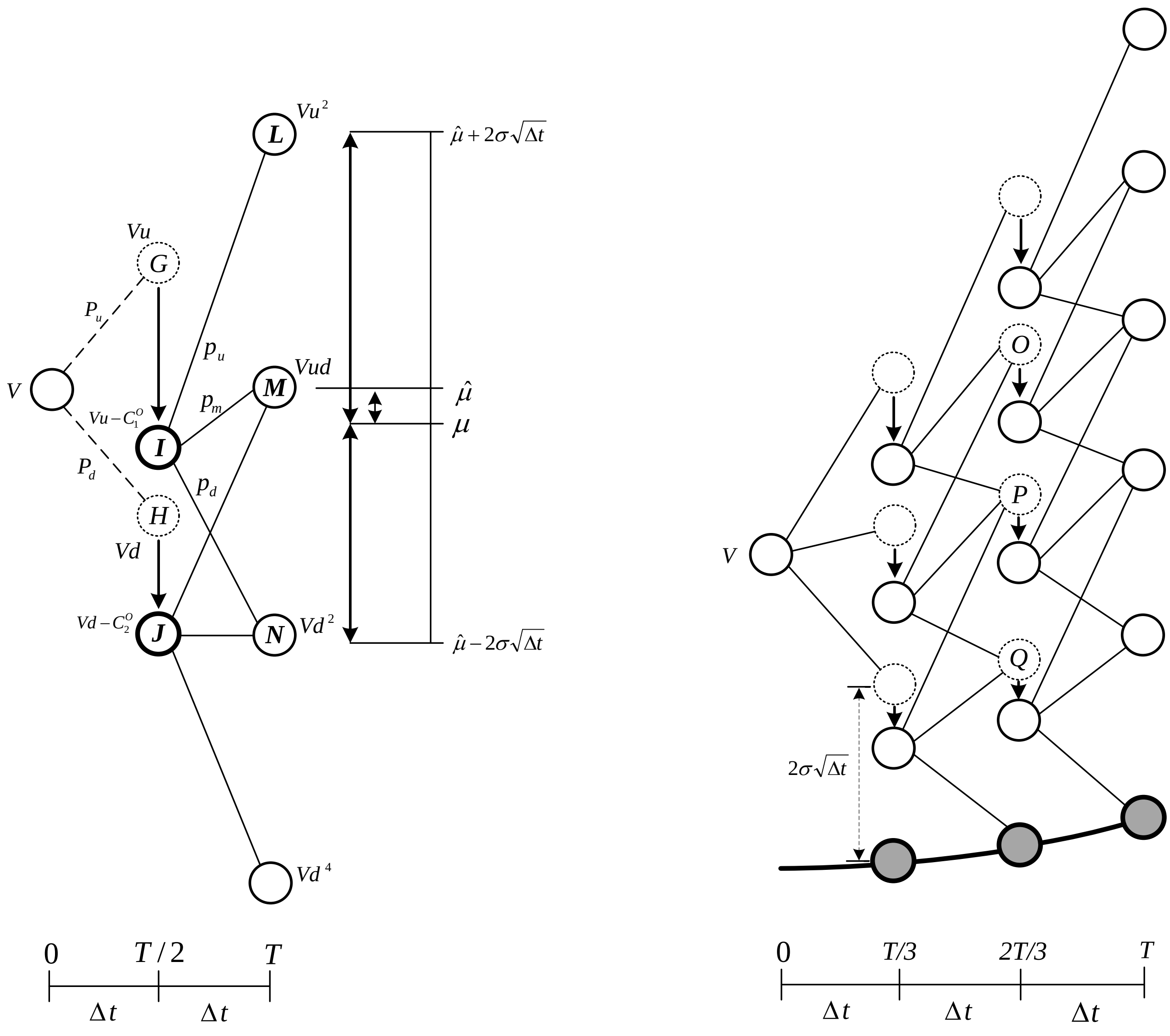
where $t = T/2$, $V_{t^+} = v(I)$, $V^u_{(t+\Delta t)^-} = v(L)$, $V^m_{(t+\Delta t)^-} = v(M)$, and $V^d_{(t+\Delta t)^-} = v(N)$, respectively.

In addition to the node proliferation problem, Dai and Lyuu (2010)'s trinomial structure can also alleviate the price oscillation problem when the default condition **(D.2)** is adopted (see Figlewski and Gao, 1999). **Figure 1(b)** illustrates an example for the presence of a time-varying default boundary as a thick curve. For each time step, the gray node is first placed on the boundary to reduce the nonlinearity error, and all other nodes like node O, P, and Q at time $t = 2T/3$ are then laid out from the gray node upward. The log-distance between any two vertically adjacent nodes remains $2\sigma\sqrt{\Delta t}$. Under this layout, Dai and Lyuu (2010)'s trinomial branches allow us to connect one node at any time step to its successor nodes at the next time step with suitable branching probabilities no matter what payout policy is adopted.

The trinomial structure can also be applied to the otherwise identical process of **Equation (1)** under the real world probability measure **Q**. The corresponding real world branching probabilities q_u, q_m, and q_d can be acquired by on the other hand matching $\overline{\mu} = (\eta - \sigma^2/2)\Delta t$ and `Var`, respectively.

2.2.2 Forests for Transitions of Capital Structure

Capital injection into a firm such as new share issuance or capital outflow such as debt repayment results in transitions of capital structure and thus influences the values of the firm's claim holders. The forest structure proposed by Liu et al. (2016) allows us to evaluate the contingent claims considering these transitions. A forest consists of several trees with different root nodes to capture the firm value dynamics with different initial values. Each tree evaluates the contingent claims under one state of capital structure, say before and after capital injection, via the backward induction on that tree. Once



(a) (b)

Figure 1: Binomial and trinomial tree structure. Panel (a) illustrates a tree structure for the evolvement of a firm's unlevered asset value when the firm has cash payments C_1^O and C_2^O at time $t = T/2$. The dashed branches are the CRR binomial structure with the parameters u, d, P_u and P_d. u and d govern the state of the asset value; P_u and P_d are the branching probabilities of the asset value to move up and down. The solid branches are Dai and Lyuu (2010)'s trinomial structure with the risk-neutral branching probabilities p_u, p_m and p_d that are determined by asymptotically matching μ and `Var`. This trinomial structure can avoid the node proliferation problem due to the presence of discontinuous jumps. Panel (b) illustrates that these trinomial branches can also alleviate the price oscillation problem, because they allows us to connect one node at any time step to its successor nodes laid upward from the gray node placed on the time-varying default boundary at the next time step.

the backward induction is made from one tree to another, the effect of capital structure change on the contingent claim values is taken into account.

Now we describe a generic example for the presence of capital injection due to new share issuance.[2]

[2] Liu et al. (2016) provide another generic example for capital outflows and thus changes of capital structures due to

The resulting forest illustrated in **Figure 2** is composed of three otherwise identical trees that are placed in different layers to capture three firm value dynamics with different initial values $v(A) < v(B) < v(C)$. The tree with the highest root node C is placed on the highest layer, whereas the one with lowest root node A is placed on the lowest layer. Each tree captures one state of capital structure. For ease of illustration, we assume that the firm has no cash payout during the time interval $[0, T]$. All of the three trees include the same time-varying default boundary as a thick curve. To alleviate the price oscillation problem, these trees are constructed to have the gray nodes that are placed on the boundary as **Figure 1(b)**. The solid trinomial branches in the forest are Dai and Lyuu (2010)'s trinomial structure. These branches not only connect the nodes that are laid out from the gray nodes upward in the same tree but to connect the nodes in different trees on different layers to reveal transitions of capital structure. For example, the firm will issue new shares to acquire capital C_t^I on node J at time $t = T/4$ and on node U at time $t = 3T/4$. The node U then jumps upward to node W to reveal the capital injection into the equity-issuing firm, and $v(W) = v(U) + C_{3T/4}^I$. The branching probabilities of this trinomial structure are determined via the procedure discussed in Section 2.2.1. The discounted expected values of the firm's contingent claims on node W in the middle-layer tree is calculated from the values on nodes X, Y, and Z in the highest-layer tree via the **Equation (4)**. The discounted expected values of the contingent claims on node L at time $t = T/4$ can also be calculated in the same way.

3 A State-Transition Tree Allowing for Sequential Exercise Policy

In this section, a state-transition tree is developed to systematically incorporate sequential exercise policy following share dilution and capital injection (if existent). This tree can be applied not only to risk-neutral valuation of tradable assets but also to utility-based valuation of nontransferable assets. For tradable assets, we take the evaluation of a `CB` as an example in Section 3.1. For nontransferable assets, we take the evaluation of `ESOs` as an example in Section 3.2. For ease of illustration, we first introduce this tree structure by describing the backward induction procedure on that tree. Then, we visualize the tree.

3.1 A Risk-Neutral Valuation of a Tradable `CB`

3.1.1 Pricing with Backward Induction

Notice that the `SB` and callable `CB` specified in **Equation (2)** are both mature at time T. We denote the time ν, $0 < \nu \leq T$, such that the call protection period for the `CB` is $[0, \nu)$. Once $\nu = T$, the callable `CB` degenerates into a noncallable `CB`. The backward induction procedure on a n-step state-transition tree, $\Delta t = T/n$, starts from time T and is separated into three parts: $t = T$, $\nu \leq t < T$, and $0 \leq t < \nu$.

Part 1: $t = T$

Assume that the `CB` is not called and $x\%$ of its face value has been converted before time t^-. The `CB` holder decides to convert the `CB` to $y\%$ at time t^-, $0 \leq x \leq y \leq 100$. The firm will repay the `SB` and the remaining `CB` at their maturity date if the firm is solvent at time t^-. Otherwise, the firm announces default and files for bankruptcy. The equity value can thus be expressed as

$$E_{NC}(t, V_t, x\%, y\%) = \begin{cases} V_t - Pay_{NC}(t, x\%, y\%) & \text{if default does not occur at } t^-, \\ 0 & \text{otherwise,} \end{cases} \tag{5}$$

premature redemption of callable bonds.

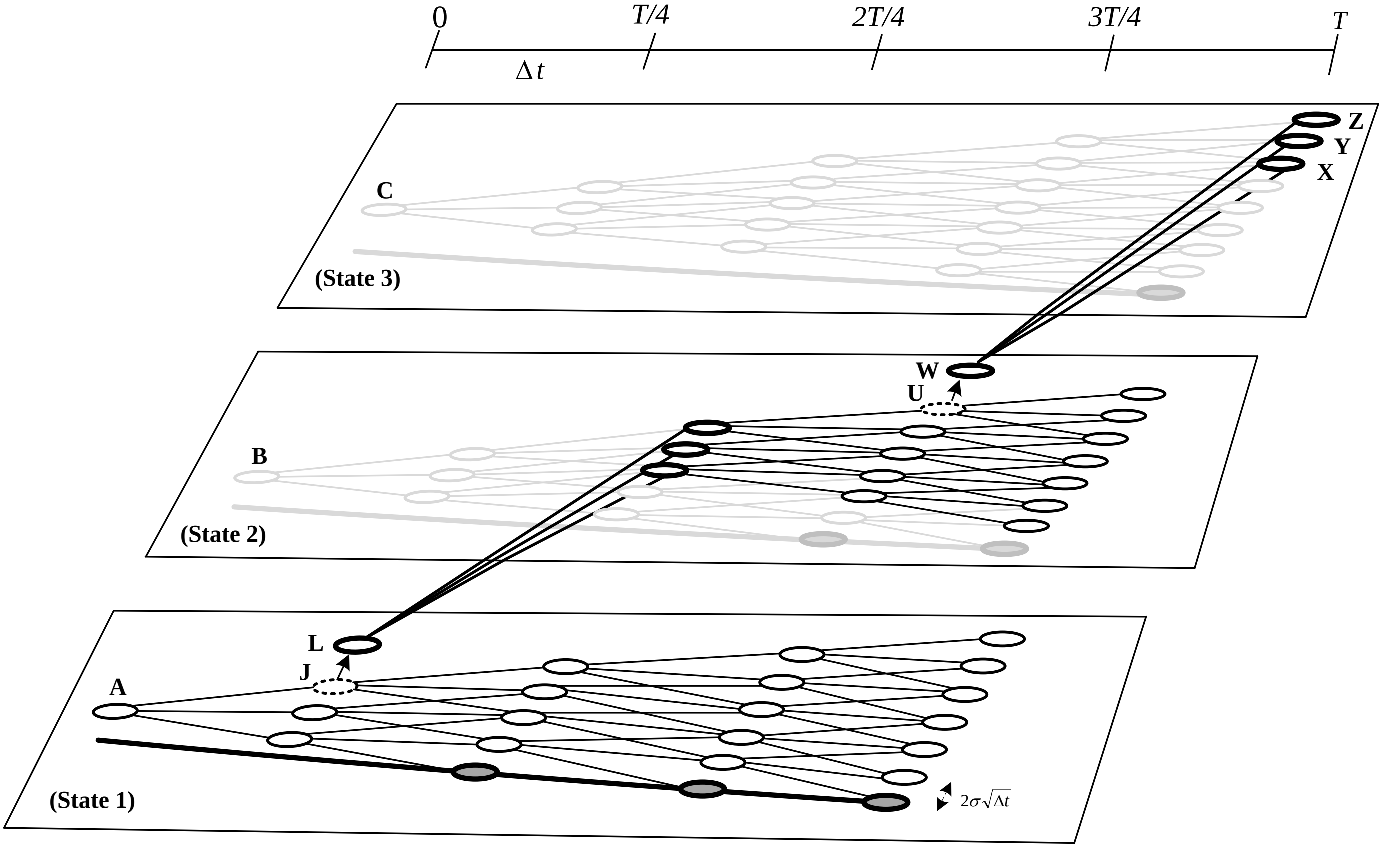


Figure 2: A Forest for Transitions of Capital Structure due to Capital Injection. This figure illustrates a forest comprised by three otherwise identical trees with different root nodes A, B, and C. These root nodes refer to different initial firm's asset values $v(A) < v(B) < v(C)$. Each tree in the forest captures one state of capital structure, say before or after capital injection. All of the three trees include the same time-varying default boundary as a thick curve, and they are constructed to have the gray nodes that are placed on the boundary to alleviate the price oscillation problem. The solid trinomial branches are the Dai and Lyuu (2010)'s trinomial structure with suitable branching probabilities as **Figure 1(a)**. They not only connect the nodes that are laid out from the gray nodes upward in the same tree but to connect the nodes in different trees on different layers to reveal transitions of capital structure.

where

$$Pay_{NC}(t, x\%, y\%) = \underbrace{F_s(1 + (1-\tau)C_s\Delta t)}_{\textbf{A}} + \underbrace{(1-y\%)F_c + (1-x\%)F_c(1-\tau)C_c\Delta t}_{\textbf{B}}.$$

Part A represents the after-tax payments to the SB, and Part B refers to the payments including the accrued interest to the remaining CB.[3] The default state indicates the timing when $V_{t^-} < Pay_{NC}(t, x\%, x\%)$, for we suppose in Section 2.1.3 that default occurs before the conversion decision. Associated with the equity value, the remaining CB value is

$$CB_{NC}(t, V_t, x\%, y\%)$$
$$= \begin{cases} (1-y\%)F_c(1+C_c\Delta t) + (y\% - x\%)\underbrace{(\lambda S_{NC}(t, V_t, y\%) + F_cC_c\Delta t)}_{\textbf{C}}, & \text{if default does not occur at } t^-, \\ max((1-\omega)V_t - F_s, 0), & \text{otherwise,} \end{cases} \tag{6}$$

[3] Although accrued interest is not always paid when a CB is converted, Bhattarcharya (2012) mentions that there has been an increasing tendency to pay accrued interest in recent years.

where

$$S_{NC}(t, V_t, y\%) = \frac{E_{NC}(t, V_t, x\%, y\%)}{N + \underbrace{y\%\lambda}_{\mathbf{D}}}. \tag{7}$$

Part C of **Equation (6)** indicates the conversion value plus accrued interest. The $S_{NC}(t, V_t, y\%)$ is the share price after $y\%$ of the `CB` are converted, and it can be calculated via **Equation (7)**. Part D of this equation represents the increment in the shares of common stocks due to the conversion that causes share dilution.

The parameters $x\%$ and $y\%$ characterize sequential conversion policies, and a block conversion is merely one special case of them (i.e., when $x = 0$ and $y = 100$). A risk-neutral `CB` holder's utility can be maximized by directly maximizing the present value of her `CB`. Thus, she will formulate a conversion policy $y = \alpha(t^-, V_{t^-})$ at time t^-, $x \le \alpha(t^-, V_{t^-}) \le 100$, such that

$$\alpha(t^-, V_{t^-}) = \begin{cases} \arg\max_y CB_{NC}(t^-, V_{t^-}, x\%, y\%) & \text{if default does not occur at } t^-, \\ x & \text{otherwise.} \end{cases} \tag{8}$$

Finally, the `SB` value given the conversion policy $\alpha(t^-, V_{t^-})$ is

$$SB_{NC}(t, V_t, x\%, \alpha(t^-, V_{t^-})\%) = \begin{cases} F_s(1 + C_s\Delta t) & \text{if default does not occur at } t^-, \\ min(F_s, (1-\omega)V_t) & \text{otherwise.} \end{cases} \tag{9}$$

Note that the second equation in **Equations (6)** and **(9)** stand for the liquidation process according to the absolute priority rule. Once the firm is bankrupt, the remaining asset will be first distributed to the senior `SB` holder and then to the junior `CB` holder.

On the other hand, if the `CB` is called and $y\%$ of the bond is converted before t^-, then the equity value is

$$E_C(t, V_t, y\%, y\%) = \begin{cases} V_t - F_s(1 + (1-\tau)C_s\Delta t) & \text{if } V_{t^-} \ge F_s(1 + (1-\tau)C_s\Delta t), \\ 0 & \text{otherwise,} \end{cases} \tag{10}$$

and the share price $S_C(t, V_t, y\%)$ can be evaluated by replacing the numerator in **Equation (7)** with $E_C(t, V_t, y\%, y\%)$. In addition,

$$SB_C(t, V_t, y\%, y\%) = \begin{cases} F_s(1 + C_s\Delta t) & \text{if } V_{t^-} \ge F_s(1 + (1-\tau)C_s\Delta t), \\ min(F_s, (1-\omega)V_t) & \text{otherwise.} \end{cases} \tag{11}$$

Part 2: $\nu \le t < T$

Consider the time t and $t + \Delta t$, and $t + \Delta t \le T$. Let t^- and t^+ denote the times immediately before and after a payout. A contingent claim value at time t^- thus depends on its value at time t^+. If the `CB` is not called and $y\%$ of the `CB` is converted at time t^-, then the equity value is expressed as

$$\begin{aligned} & E_{NC}(t^-, V_{t^-}, x\%, y\%) \\ &= \begin{cases} (N + y\%\lambda) \cdot S_{NC}(t^+, V_{t^+}, y\%) - Pay_{NC}(t, x\%, y\%) & \text{if default does not occur at } t^-, \\ 0 & \text{otherwise.} \end{cases} \end{aligned} \tag{12}$$

$S_{NC}(t^+, V_{t^+}, y\%)$ is calculated via **Equation (4)** given that the conversion policy at time $(t + \Delta t)^-$ is $\alpha\left((t+\Delta t)^-, V_{(t+\Delta t)^-}\right)$, $y \le \alpha\left((t+\Delta t)^-, V_{(t+\Delta t)^-}\right) \le 100$. This implies that the share price reflects future share dilution due to conversion. In addition, $Pay_{NC}(t, x\%, y\%)$ depends on the payout policy specified in Section 2.1.2. In summary,

(P.1) $Pay_{NC}(t, x\%, y\%) = F_s(1-\tau)C_s\Delta t + (1-x\%)F_c(1-\tau)C_c\Delta t - \theta_t$; $V_{t^+} = V_{t^-} - \theta_t$;

(P.2) $Pay_{NC}(t, x\%, y\%) = 0$; $V_{t^+} = V_{t^-} - F_s(1-\tau)C_s\Delta t - (1-x\%)F_c(1-\tau)C_c\Delta t$,

where $\theta_t = V_{t^-}e^{\delta\Delta t} - V_{t^-}$ by following Broadie and Kaya (2007). On the other hand, the default trigger could be either **(D.1)** or **(D.2)** specified in Section 2.1.3. In particular, with **(D.1)**, the firm determines to file for bankruptcy once $E_{NC}(t^-, V_{t^-}, x\%, x\%) = 0$, which implies that the default decision is always made prior to the conversion decision. Similarly, with **(D.2)**, the firm is forced to file for bankruptcy before the conversion decision once V_{t^-} is equal to or less than the exogenous default boundary D_{t^-}.

If the CB is called at time t^-, the equity value is on the other hand expressed as

$$E_C(t^-, V_{t^-}, x\%, y\%) = (N + y\%\lambda) \cdot S_C(t^+, V_{t^+}, y\%) - Pay_C(t, x\%, y\%), \tag{13}$$

where the $Pay_C(t, x\%, y\%)$ is

(P.1) $Pay_C(t, x\%, y\%) = Pay_{NC}(t, x\%, y\%) + \underbrace{(1-y\%)K_c}_{\mathbf{E}}$; $V_{t^+} = V_{t^-} - \theta_t$.

(P.2) $Pay_C(t, x\%, y\%) = 0$; $V_{t^+} = V_{t^-} - F_s(1-\tau)C_s\Delta t - (1-x\%)F_c(1-\tau)C_c\Delta t - \underbrace{(1-y\%)K_c}_{\mathbf{E}}$.

Part E reveals the repayment once the firm redeems the remaining CB when the converted portion increases from $x\%$ to $y\%$. Via **Equation (4)**, the $S_C(t^+, V_{t^+}, y\%)$ is again calculated based on its value at time $(t+\Delta t)^-$. Finally, with the conversion policy $y\%$ given $x\%$ at time t^-, the firm will choose its call policy to maximize its equity holders' value. That is,

$$E(t^-, V_{t^-}, x\%, y\%) = max\left[E_{NC}(t^-, V_{t^-}, x\%, y\%), E_C(t^-, V_{t^-}, x\%, y\%)\right]. \tag{14}$$

Then, $S(t^-, V_{t^-}, y\%)$ is obtained by replacing the numerator in **Equation (7)** with above $E(t^-, V_{t^-}, x\%, y\%)$.

The remaining CB value associated with the call policy in **Equation (14)** is expressed as follows. If the firm does not announce call at time t^-, then

$$CB_{NC}(t^-, V_{t^-}, x\%, y\%) = \begin{cases} CB_{NC}(t^+, V_{t^+}, x\%, y\%) + (1-x\%)F_cC_c\Delta t + (y\% - x\%)\,\lambda S(t^-, V_{t^-}, y\%), & \text{if default does not occur at } t^-. \\ max((1-\omega)V_{t^-} - F_s, 0), & \text{otherwise.} \end{cases} \tag{15}$$

If the firm announces call, then

$$CB_C(t^-, V_{t^-}, x\%, y\%) = (1-y\%)K_c + (y\% - x\%)(\lambda S(t^-, V_{t^-}, y\%) + F_cC_c\Delta t). \tag{16}$$

Note that $y > x$ in **Equation (16)** reflects the conversion forced by call. In response to the call policy, the CB holder will formulate the conversion policy $y = \alpha(t^-, V_{t^-})$, $x \le \alpha(t^-, V_{t^-}) \le 100$, such that the remaining CB value is maximized. That is,

$$\alpha(t^-, V_{t^-}) = \begin{cases} \arg\max\limits_{y} \left\{CB_{NC}(t^-, V_{t^-}, x\%, y\%), CB_C(t^-, V_{t^-}, x\%, y\%)\right\} & \text{if default does not occur at } t^-. \\ x & \text{otherwise.} \end{cases} \tag{17}$$

The values of the E and remaining CB associated with the policy $\alpha(t^-, V_{t^-})$ can thus be expressed as $E(t^-, V_{t^-}, x\%, \alpha(t^-, V_{t^-})\%)$ and $CB(t^-, V_{t^-}, x\%, \alpha(t^-, V_{t^-})\%)$, and the SB value is

$$SB(t^-, V_{t^-}, x\%, \alpha(t^-, V_{t^-})\%) = \begin{cases} SB(t^+, V_{t^+}, x\%, \alpha(t^-, V_{t^-})\%) + F_sC_s\Delta t & \text{if default does not occur at } t^-. \\ min(F_s, (1-\omega)V_{t^-}) & \text{otherwise.} \end{cases} \tag{18}$$

Part 3: $0 \le t < \nu$

First, we consider $0 < t < \nu$. The equity value is as **Equation (12)**, since premature redemption is not allowed. The corresponding CB value can also be expressed as **Equation (15)**. Given that $x\%$ of the CB is converted before t^-, the CB holder can always find a conversion policy $y = \alpha(t^-, V_{t^-})$, $x \leq \alpha(t^-, V_{t^-}) \leq 100$, as **Equation (8)**. The E and remaining CB values are thus expressed to associate with the policy $\alpha(t^-, V_{t^-})$. The SB value can be expressed as **Equation (18)**. Finally at $t = 0$, we set $x = y = 0$. Then, $E(0, V_0, 0\%, 0\%) \equiv E(0, V_0)$, $S(0, V_0, 0\%) \equiv S(0, V_0)$, $CB(0, V_0, 0\%, 0\%) \equiv CB(0, V_0)$, and $SB(0, V_0, 0\%, 0\%) \equiv SB(0, V_0)$ as those in **Equation (2)**.

3.1.2 Visualization of a State-Transition Tree

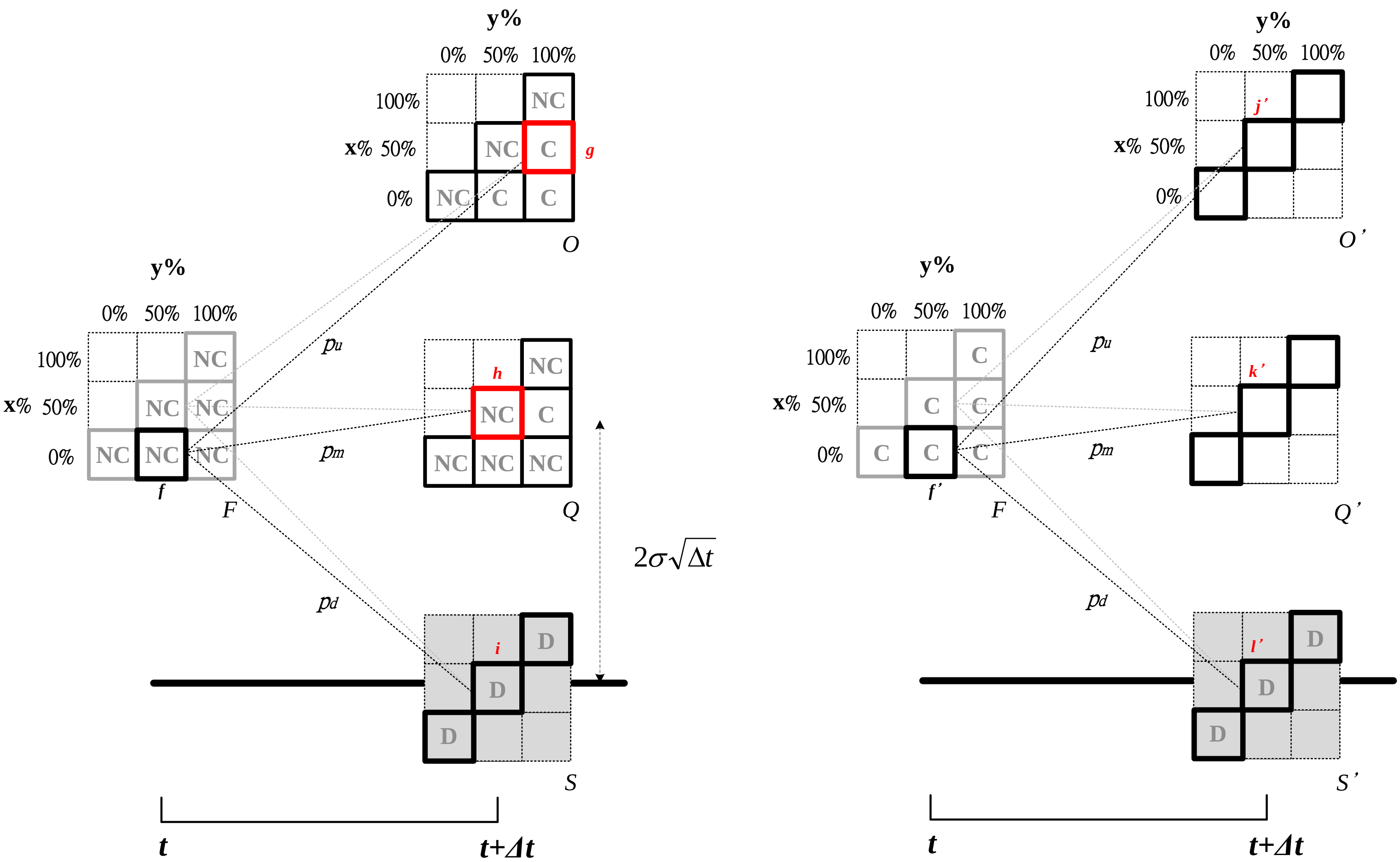
The backward induction procedure in Section 3.1.1 can be implemented via two state-transition trees, and this procedure from time $t + \Delta t$ to t is visualized in **Figure 3**. This figure displays a one-period trinomial structure similar to any one in **Figure 1(b)** with a gray asset value node placed on the default boundary. The risk-neural branching probabilities p_u, p_m, and p_d can also be determined via the procedure in Section 2.2.1. The main difference between the trees in Section 2.2 and a state-transition tree is that each asset value node on the latter tree consists of several dot and solid squares. Those solid squares allow us to capture one state of sequential exercise conditions and reveal the diminishing outstanding CB due to conversion.

In **Figure 3(a)**, each solid square at time $t + \Delta t$ stands for the scenario that the CB holder determines to convert the CB to $y\%$ at time $(t + \Delta t)^-$ given that the CB is not prematurely redeemed and $x\%$ of the bond have already converted at time t^+. The condition $0 \leq x \leq y \leq 100$ makes those solid squares cluster in the diagonal and lower triangle parts of each asset value node if the CB issuer is solvent. The x and y could be 0, 50, or 100 in **Figure 3**. More possible values of x and y can be designated if we add more squares to each asset value node. The gray node placed on the default boundary contains only three solid squares located in the diagonal part (i.e., $x = y$). It reveals that default occurs prior to any conversion decision. For clarification, mark "D" is put into the black solid squares to denote the state "Default". To capture the call policy specified by **Equation (14)**, the "C" in each solid square at time $(t + \Delta t)^-$ indicates that the best policy for the CB issuer at time $(t + \Delta t)^-$ is to announce call, whereas the "NC" indicates the opposite. If $t + \Delta t$ is in the call protection period, all of the call policies in solid squares are "NC" to reveal prohibition of early redemption.

Figure 3(a) illustrates the case that the issuer decides not to announce call at time t^-. Conditional on the call decisions subject to every conversion policy $y\%$ given $x\%$ at time $(t + \Delta t)^-$, the red solid squares represent the optimal conversion policies $y = \alpha\left((t + \Delta t)^-, V_{(t+\Delta t)^-}\right)$, $x \leq \alpha((t+\Delta t)^-, V_{(t+\Delta t)^-}) \leq 100$, that are specified by **Equations (8)** or **(17)**. Associated with the branching probabilities, the E, CB, and SB values in these red squares at time $(t + \Delta t)^-$ are used to calculate the discounted expected values of the three contingent claims in each solid square at t^+ as **Equation (4)**. For example, the discounted expected values in the black solid square f at time t^+ are calculated based on the values in the red solid square g and h and black solid square i at time $(t + \Delta t)^-$, whose x equals to the y of the f to reflect the additional conversion amount across the time period. The E, CB, and SB values in f at time t^- are then obtained by considering the payout at time t. Finally for clarification, we attach the mark "NC" to f and other solid squares to represent the contingent claim values at time t^- when the state at that time is "Not Called".

Figure 3(b) illustrates the case that the CB will no longer exist at time $(t + \Delta t)^-$ because the issuer decides to call the remaining $(1 - y\%)$ of the CB back at time t^-. The black solid square at time $(t + \Delta t)^-$ will locate in the diagonal part of each asset value node to reflect that the CB is removed and thus no other call and conversion decisions hereafter. The `E` and `SB` values in each black solid square at time $(t + \Delta t)^-$ are expressed as **Equations (10)** and **(11)** and are used to calculate their discounted expected values in each solid square at time t^+ as **Equation (4)**. For example, the discounted expected values in black solid square f' at time t^+ are calculated according to the values in j', k' and l' at time $(t + \Delta t)^-$, whose x equals the y of the f' to reflect no more conversion volumes after t^+. The `E` and `SB` values in f' at time t^- are then obtained by considering the payout at time t. The corresponding `CB` value is on the other hand calculated via **Equation (16)**. Finally, we attach the mark "C" to f' and other solid squares to represent the contingent claim values at time t^- when the state at that time is "Called".

To reflect the call policies subject to every conversion policy at time t^-, the final `E`, `CB`, and `SB` values in each solid square within the node F will be chosen based on **Equation (14)**. Like the cases within node O or Q in **Figure 3(a)**, we plug "NC" in a solid square within node F if the chosen values are those with "NC" and plug "C" for the opposite. Finally, the red solid squares in each row of solid squares will again be chosen based on **Equation (8)** or **(17)**, and those are used to implemented another backward induction from time t^- to $(t - \Delta t)^+$.



(a) Call is not announced at time t^-

(b) Call is announced at time t^-

Figure 3: The Backward Induction Procedure and One-Period State-Transition Trees. This figure displays the backward induction procedure via a one-period state-transition tree from time $t + \Delta t$ to t. The dot trinomial branches are the Dai and Lyuu (2010)'s trinomial structure with branching probabilities p_u, p_m, and p_d as those in **Figure 1(a)**. Panel (a) illustrates the state that call is not announced at time t^-. In this panel, each solid square at time $(t + \Delta t)^-$ contains the values of the `E`, `SB`, and remaining `CB` when the `CB` is converted to y% at time $(t + \Delta t)^-$ given that x% of the bond is converted until time t^+, $0 \le x \le y \le 100$. In each solid square that characterizes the conversion policy y% given x% at time $(t + \Delta t)^-$, "C" indicates that the `CB` issuer's best policy is to announce call, whereas "NC" indicates the opposite. Among these solid squares at time $(t + \Delta t)^-$, the red solid ones represent the optimal conversion policy $y = \alpha((t + \Delta t)^-, V_{(t+\Delta t)^-})$ given x, $x \le \alpha((t+\Delta t)^-, V_{(t+\Delta t)^-}) \le 100$. The discounted expected values of the `E`, `SB`, and the remaining `CB` in each solid square at time t^- are calculated based on the their values in the red solid squares at time $(t + \Delta t)^-$, the risk-neutral branching probabilities, and the payout occurred at time t. These are the contingent claim values at time t^- with mark "NC". Panel (b) illustrates the state that call is announced at time t^-. In this panel, the solid squares at time $(t + \Delta t)^-$ locate in the diagonal part to reveal no other call and conversion decisions. The contingent claim values in these solid squares associated with the payout occurred at time t are used to calculate the contingent claim values in every solid square at time t^- with mark "C". The gray node placed on the boundary represents the default state. The solid squares with mark "D" locate in the diagonal part to reveal that default occurs prior to any conversion decision. Notice that the amounts of sequential conversion are equally separated into 50% of the `CB` here. The partition can be set to any percentage of the `CB` if more squares are added to each asset value node.

3.2 A Utility-Based Valuation of Nontransferable ESOs

3.2.1 Pricing with Backward Induction

In the same token, we denote the time ν, $0 < \nu \leq T$, such that the vesting period for the T-year ESO is $[0, \nu)$. The backward induction procedure again starts from time T and is also separated into three parts: $t = T$, $\nu \leq t < T$, and $0 \leq t < \nu$.

Part 1: $t = T$

Assume that $x\%$ of the total ESOs have been exercised before time t^-. The representative ESO holder decides to exercise these ESOs to $y\%$ at time t^-, $0 \leq x \leq y \leq 100$. The firm will repay the SB at its maturity date if the firm is solvent. Otherwise, the firm announces default and files for bankruptcy. The equity value, $E(t, V_t, x\%, y\%)$, can then be expressed as **Equation (5)** but the payout is

$$Pay(t, x\%, y\%) = F_s(1 + (1-\tau)C_s\Delta t) \underbrace{-(y\% - x\%)N_cK_c}_{\mathbf{F}},$$

where the minus term Part F reflects the capital injection into the firm because additional ESOs are exercised at time t^-. Since the firm issues additional new shares for the ESO holder, the share price is as **Equation (7)** with $E(t, V_t, x\%, y\%)$ as the numerator, but the $\lambda = 1$ to on the other hand reflects share dilution. Associated with the share price, the exercise payoff of one ESO is

$$Payoff(t^-, V_{t^-}, x\%, y\%) = \left(S(t^-, V_{t^-}, y\%) - K_c\right)^+. \tag{19}$$

Once the whole ESOs are exercised from $x\%$ to $y\%$, the holder has the utility of the exercise payoff

$$\begin{aligned} TU(t^-, V_{t^-}, x\%, y\%) &= U\Big(t^-, Payoff(t^-, V_{t^-}, x\%, y\%)\Big) \\ &= \Big[(y\% - x\%) \cdot N_c \cdot Payoff(t^-, V_{t^-}, x\%, y\%)\Big]^\gamma \end{aligned}$$

according to **Equation (3)**. To maximize the utility of this payoff, the ESO holder will formulate the policy $y = \alpha(t^-, V_{t^-})$ at time t^-, $x \leq \alpha(t^-, V_{t^-}) \leq 100$, such that

$$\alpha(t^-, V_{t^-}) = \arg\max_y TU(t^-, V_{t^-}, x\%, y\%). \tag{20}$$

The total ESO value can thus be expressed as

$$ESO\left(t^-, V_{t^-}, x\%, \alpha(t^-, V_{t^-})\%\right) = \left(\alpha(t^-, V_{t^-})\% - x\%\right) \cdot N_c \cdot Payoff(t^-, V_{t^-}, x\%, \alpha(t^-, V_{t^-})\%), \tag{21}$$

and the SB value subject to the policy $\alpha(t^-, V_{t^-})$ is as **Equation (9)**.

Part 2: $\nu \leq t < T$

Consider the time t and $t + \Delta t$, and $t + \Delta t \leq T$. Again, let t^- and t^+ denote the times immediately before and after a payout, and a contingent claim value at time t^- depends on its value at time t^+. If $y\%$ of the total ESOs are exercised at time t^-, the equity value is expressed as

$$\begin{aligned} &E(t^-, V_{t^-}, x\%, y\%) \\ &= \begin{cases} (N + y\%N_c) \cdot S(t^+, V_{t^+}, y\%) - Pay(t, x\%, y\%) & \text{if default does not occur at } t^-, \\ 0 & \text{otherwise,} \end{cases} \end{aligned} \tag{22}$$

where $Pay(t, x\%, y\%)$ depends on the payout policy in Section 2.1.2. In summary

(P.1) $Pay(t, x\%, y\%) = N_sF_s(1-\tau)C_s\Delta t - \theta_t$; $V_{t^+} = V_{t^-} \underbrace{+(y\% - x\%)N_cK_c}_{\mathbf{H}} - \theta_t$;

$$\textbf{(P.2)} \quad Pay(t, x\%, y\%) = 0 \; ; \; V_{t^+} = V_{t^-} \underbrace{+(y\% - x\%)N_c K_c}_{\textbf{H}} - N_s F_s (1-\tau) C_s \Delta t,$$

where the plus term part H reflects the capital injection due to the exercise of additional ESOs. The exercise payoff of one ESO at time t^- is as **Equation (19)**. If the whole ESOs are exercised from $x\%$ to $y\%$ at time t^-, the holder has the expected utility of the intertemporal payoffs occurred since t^-

$$\begin{aligned}
& TU(t^-, V_{t^-}, x\%, y\%) \\
&= U\Big(t^-, \, Payoff(t^-, V_{t^-}, x\%, y\%)\Big) + TU\Big(t^+, \, V_{t^+}, y\%, \alpha((t+\Delta t)^-, V_{(t+\Delta t)^-})\%\Big) \\
&= \Big[(y\% - x\%) \cdot N_c \cdot Payoff\left(t^-, V_{t^-}, x\%, y\%\right)\Big]^{\gamma} + TU\Big(t^+, \, V_{t^+}, y\%, \alpha\left((t+\Delta t)^-, V_{(t+\Delta t)^-}\right)\%\Big),
\end{aligned} \tag{23}$$

where

$$\begin{aligned}
& TU\Big(t^+, \, V_{t^+}, y\%, \alpha((t+\Delta t)^-, V_{(t+\Delta t)^-})\%\Big) \\
&= e^{-\beta\Delta t}\Big[q_u \cdot TU\left((t+\Delta t)^-, \, V^u_{(t+\Delta t)^-}, y\%, \alpha\left((t+\Delta t)^-, V^u_{(t+\Delta t)^-}\right)\%\right) + \\
& \qquad q_m \cdot TU\left((t+\Delta t)^-, \, V^m_{(t+\Delta t)^-}, y\%, \alpha\left((t+\Delta t)^-, V^m_{(t+\Delta t)^-}\right)\%\right) + \\
& \qquad q_d \cdot TU\left((t+\Delta t)^-, \, V^d_{(t+\Delta t)^-}, y\%, \alpha\left((t+\Delta t)^-, V^d_{(t+\Delta t)^-}\right)\%\right)\Big].
\end{aligned}$$

The above equation is the discounted expected utility due to intertemporal exercises of additional ESOs after time t^+, and it is calculated under the real world probability measure $\mathbf{Q}$ as discussed in Section 2.1.4. To maximize the expected utility, the ESO holder will formulate the policy $y = \alpha(t^-, V_{t^-})$ at time t^- as **Equation (20)**. From the perspective of the ESO issuer, the ESO value is then expressed as

$$\begin{aligned}
& ESO\left(t^-, V_{t^-}, x\%, \alpha(t^-, V_{t^-})\%\right) \\
&= \underbrace{ESO\left(t^+, V_{t^+}, \alpha(t^-, V_{t^-})\%, \alpha\left((t+\Delta t)^-, V_{(t+\Delta t)^-}\right)\%\right)}_{\textbf{I}} + \underbrace{\left(\alpha(t^-, V_{t^-})\% - x\%\right) \cdot N_c \cdot Payoff\left(t^-, V_{t^-}, x\%, \alpha(T^-, V_{T^-})\%\right)}_{\textbf{J}},
\end{aligned} \tag{24}$$

where the part I is the value of the remaining unexercised ESOs at time t^+ and is evaluated via **Equation (4)** under the risk-neutral probability measure $\mathbf{P}$. The part J indicates the payoff due to the exercise of the ESOs. The total SB value is as **Equation (18)**. Finally, we let $x = 0$ as $t = \nu$, the time immediately after the end of the vesting period.

Part 3: $0 \le t < \nu$

Within the vesting period, both x and y are equal to 0 to reflect prohibition of exercising any ESOs. All of the contingent claim values at time t^- are evaluated by following **Equation (4)** and by considering the payout at time t. Finally at $t = 0$, $E(0, V_0, 0\%, 0\%) \equiv E(0, V_0)$, $S(0, V_0, 0\%) \equiv S(0, V_0)$, $ESO(0, V_0, 0\%, 0\%) \equiv ESO(0, V_0)$, and $SB(0, V_0, 0\%, 0\%) \equiv SB(0, V_0)$ as those in **Equation (2)**.

3.2.2 Visualization of State-Transition Trees as a Forest

To further reflect the impact of intertemporal capital injection on contingent claim values, more state-transition trees with different root nodes are incorporated as a forest. **Figure 4** illustrates a three-period forest with three different state-transition trees when the x and y have three possible values 0, 50, and 100. If they have more possible values, the forest will consist of the corresponding number of trees arranged in layers. This three-period forest is otherwise identical to the one in **Figure 2**.

The main difference between the tree in **Figure 3(a)** and those in **Figure 4** is the layout of the solid squares within each asset value node. For the row of the solid squares with $x = 0$ in the latter figure, they are placed on the lowest state 1 layer to reflect the state that no capital injection occurs before time t^- and it could occur at or after t^- due to the exercises of `ESO`s. For example, the square h within the node H reflects that no `ESO`s are exercised before time $(T/3)^-$ and the `ESO` holder decides to exercise 50% of them at $(T/3)^-$. Capital injection then raises the firm's asset value from h to i. For the row of the solid squares with $x = 50$, they are on the other hand placed on the middle state 2 layer to reflect that 50% of the `ESO`s have been exercised before time t^- and more `ESO`s could be exercised at or after t^-. For example, the square j within the node J reflects that 50% of the `ESO`s have been exercised before time $(2T/3)^-$ and their holder decides to exercise 50% more at $(2T/3)^-$ to further raise the asset value from j to k. The row of the solid squares with $x = 100$ are placed on the highest state 3 layer to reflect that the `ESO`s have been entirely exercised. No other `ESO`s exist in their issuer's capital structure.

With the firm value dynamics considering the presence of sequential capital injection, all of the contingent claim values are evaluated via the backward induction across the trees in the forest. First, the `E` and `SB` values in each solid square on the highest-layer tree are evaluated via backward induction expressed as **Equation (4)** on the same tree. Second, for the contingent claim values at time t^+ in the solid squares with $x < y$ on the middle-layer tree, they are calculated from the values in the solid squares at $(t + \Delta t)^-$ on the highest-layer tree via **Equation (4)**. For example, the `E` and `SB` values in square k at time $(2T/3)^+$ are calculated from their values in the square o, p, and q at T^- on the highest-layer tree. The `ESO` value is calculated via **Equation (24)** with the part I equal to 0, for the `ESO`s no longer exist after time $(2T/3)^+$. By considering the payouts occurred at $(2T/3)$, the three contingent claim values at $(2T/3)^-$ in j are obtained. For the contingent claim values at time t^+ in the solid squares with $x = y$ on the middle-layer tree, they are calculated from their values in the red solid squares if the firm at $(t + \Delta t)^-$ is solvent or the gray solid ones if the firm is bankrupt on the same tree. That reveals that no additional `ESO`s are exercised at t^-. For example, the `E` and `SB` values at time $(2T/3)^+$ are calculated from their values in the red solid square o', p', and q' at T^- on the same tree. So does the `ESO` value according to **Equation (24)** with the part J equal to 0, for no additional `ESO`s are exercised at $(2T/3)^-$. By considering the payouts occurred at $(2T/3)$, the three contingent claim values at $(2T/3)^-$ in square j' are obtained. The red solid square in node J is then chosen to be square j rather than j' according to **Equation (20)**. Finally, all contingent claim values in the solid squares with $x < y$ on the lowest-layer tree are evaluated from the values in red and black solid squares on the middle- and highest-layer trees via **Equation (4)**, respectively. For example, the contingent claim values in the solid squares with $x = 0$ and $y = 100$ at time t^+ are calculated from the ones at $(t + \Delta t)^-$ in the black solid squares on the highest-layer tree. The values in the solid squares with $x = 0$ and $y = 50$ at t^+, like those in i, are calculated from the ones in the red solid squares at $(t + \Delta t)^-$ on the middle-layer tree. For those in the solid squares with $x = y = 0$ at t^+, they are calculated from the values in the red solid squares if the firm at $(t + \Delta t)^-$ is solvent or the gray solid squares if the firm is bankrupt on the same tree.

4 Numerical Results

The numerical analysis focuses on the impact of allowing for sequential exercise policy on `CB` and `ESO` values and how these evaluation results are different from the ones implied by block exercise policy. In particular, the risk-neutral valuation of a `CB` incorporates the payout policy **(P.1)** and default trigger **(D.1)** listed in Sections 2.1.2 and 2.1.3, whereas the utility-based valuation of the `ESO`s considers

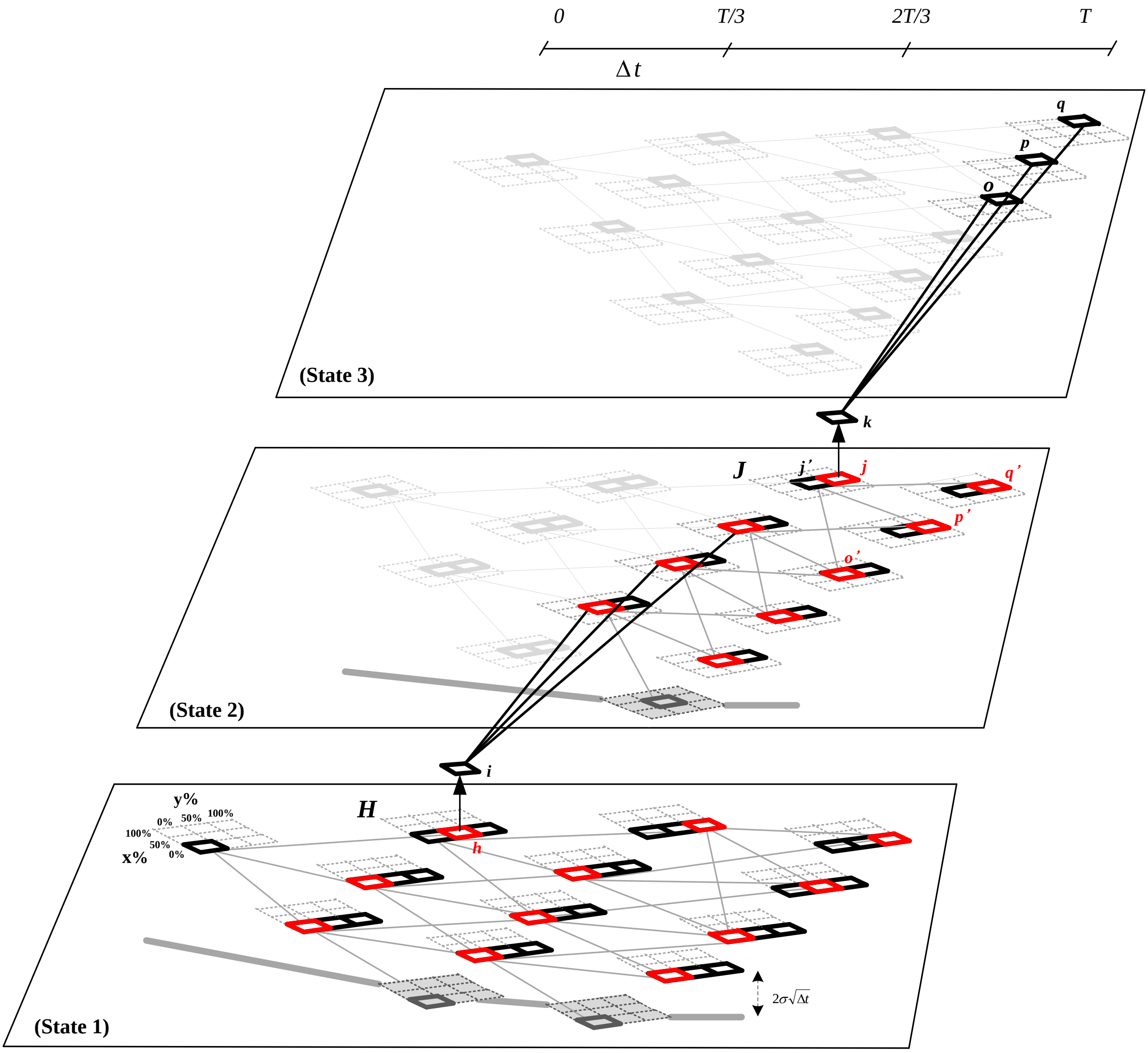


Figure 4: Three State-Transition Trees as a Forest. This figure illustrates a three-period forest comprised by three otherwise identical state-transition trees with different root nodes. Each tree in the forest captures one state of capital structure to reflect sequential capital injection due to intertemporal exercises of ESOs. This forest is otherwise identical to the one in **Figure 2**. The nodes on the lowest-layer state-transition tree only contain the row of solid squares with $x = 0$. It reflects the state that no capital injection occurs before time t^- and it could occur at or after t^- due to the exercise of ESOs. The nodes on the middle-layer state-transition tree only contain the row of solid squares with $x = 50$. It reflects that 50% of the ESOs are exercised before time t^- and more ESOs could be exercised at or after that time. The nodes on the highest-layer state-transition tree only contain the solid squares with $x = y = 100$ to reflect that ESOs are entirely exercised. The gray solid squares within the gray nodes placed on the boundary represents the default state. The red solid squares on the lowest- and middle-layer trees are chosen based on **Equation (20)**, and the contingent claim values in these red squares on the two trees are used for backward induction.

(P.2) and **(D.2)** for the comparisons confirming otherwise identical.[4] In addition, the risk-neutral

[4] The evaluation of a CB and the ESOs with exchanging settings of payout policy and default trigger is also implementable under my framework.

valuation of a callable CB or ESOs under block exercise policy is treated as the benchmark scenario. In Section 4.1, the numerical settings for the benchmark scenario are applied to all other cases, such as the valuation of a (non)callable CB under sequential conversion policy. In Section 4.2, the benchmark ESO values aim to highlight how the risk preferences of the representative employee shifts the optimal exercise policy and thus the ESO values. The exogenous default boundary specified in **(D.2)** is set to the discounted principal value of the firm's total debt as in Briys and De Varenne (1997). Other numerical settings such as the interest rate level r, the expected rate of return on the issuer's asset under the real world probability measure η, the asset value volatility σ, the payout ratio δ in **(P.1)** and the dividend policy adopted in **(P.2)**, the tax rate τ, the bankruptcy cost ω, the rate at which the representative ESO holder discount her utility of future cash flows β, and the coefficient of relative risk aversion $1-\gamma$ basically follow Jain and Subramanian (2004) and Chen et al. (2013).[5] Our evaluation framework is confirmed to produce stable pricing results in Appendix A by resolving the numeric unstability problem addressed in Figlewski and Gao (1999). The correctness of our numerical results in the following sections is also confirmed by taking advantage of the capital structure irrelevance theory proposed by Modigliani and Miller (1958).

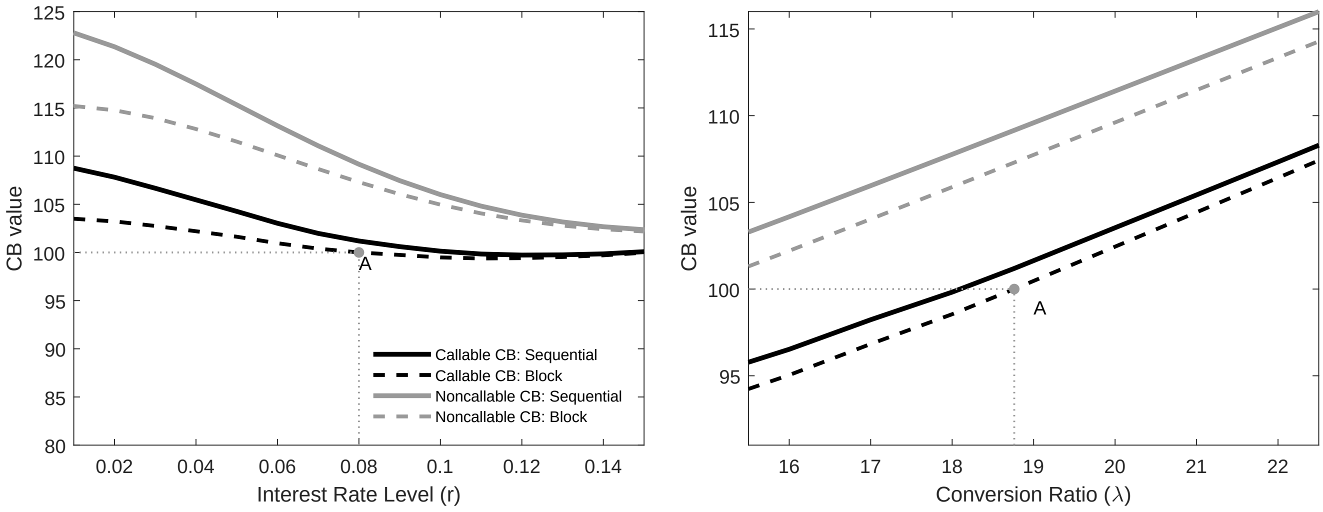
4.1 The Impacts of Sequential Conversion Policy on CB Values

Figure 5(a) illustrates the variation in callable and noncallable CB values for different choices of interest rate levels r. In general, CB values are negatively associated with the increments of r, though higher r implies greater drift terms for firm value dynamics and thus raises the stock prices to increase CB conversion values. In addition, noncallable CBs are more valuable than the callable counterparts, because the embedded call provisions grant CB issuers additional right to suppress CB values. Finally, it can be observed that CB values implied by block conversion policy can be significantly cheaper than the ones implied by sequential conversion policy. Such differences reveal the valuation impact of allowing for the latter policy as demonstrated in Emanuel (1983); Constantinides and Grundy (1987); Spatt and Sterbenz (1988).

One of the possible rationales behind sequential conversions is that CB holders need to trade their conversion profits off against the share dilution effect. Though CB holders can share CB issuers' upside potential profits by exercising the conversion options, the values of the newly converted shares will decrease with the increments of conversion volumes. By spreading out the conversions over time to minimize the dilution effect, the sum of conversion payoffs and thus the CB values are maximized. In **Figure 5(a)**, it can be observed that the divergence between solid and dashed curves is salient when r is low, because the CB values implied by the true optimal conversion policy are significantly different from those implied by the block one. However, the two policies are getting similar with the increments of r, because CB holders are prone to cluster their conversions when the underlying share prices and thus the conversion values are great in the high interest rate environments.

The valuation impact of allowing for intertemporal conversions is less salient when CBs are callable, because the presence of call provisions could force CB holders to cluster their conversions at suboptimal timings for avoidance of (impending) calls that curtail conversion values. Due to bondholder-stockholder conflicts of interest, the call policy to maximize equity holders' benefits thus shrinks the differences between the true and block conversion policies. The smaller valuation impact keeps existing even for different choices of conversion ratios λ, asset value volatility σ, and payout ratios δ as in

[5] Specifically in Jain and Subramanian (2004), they assume no dividend payments to ensure that the ESO values are implied by the no early exercise policy under the risk-neutral valuation framework.

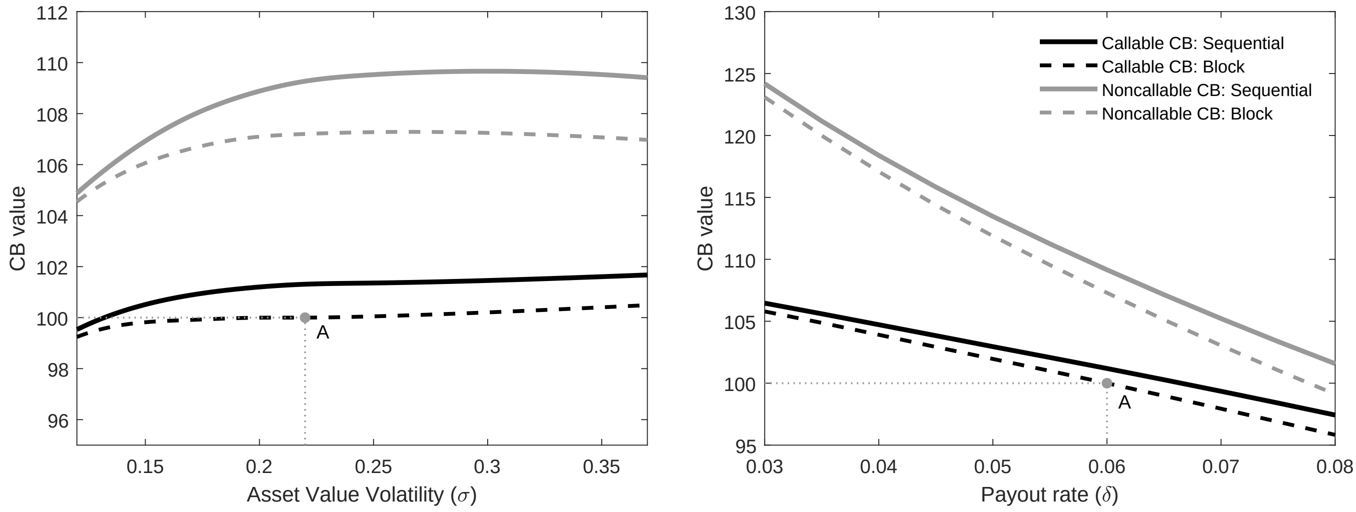


(a) (b)

Figure 5: The CB Values under Different Interest Rate Levels and Conversion Ratios. In panels (a) and (b), the dashed curves illustrate the cases when CBs are evaluated by only allowing for block conversion policy, whereas the solid curves are the one considering sequential conversion policy. The black curves denote the pricing scenarios when the CBs are callable, and the gray curves denote the opposite. Specifically, the callable CB value denoted by node A is the benchmark scenario, where the interest rate level $r = 0.08$. The conversion ratio for all scenarios in this panel is set to make the benchmark callable CB value be priced at par. In panel (b), node A is the benchmark scenario identical to the one in panel (a), and $r = 0.08$ for all scenarios in this panel. Other numerical settings are as follows. The initial issuer's asset value $V_0 = 400$; The initial volume of outstanding shares $N = 50$. The face values of the SB and CB are 100; the coupon rates of the SB and CB are 0.09 and 0.07, respectively; the call price K_c of the CB is set to 100; bond maturity $T = 10$ years; $\nu = 1$ such that the call protection period in all scenarios of callable CBs is 1 year; asset value volatility is $\sigma = 0.22$; δ in the payout policy **(P.1)** is 0.06; the tax rate τ is 0.35; the bankruptcy cost ω is 0.5.

Figure 5(b) and **Figure 6**. That makes it increasingly important to allow for sequential conversion policy when CBs are evaluated, because empirical evidence shows that firms are more likely to issue noncallable CBs to cater to the preferences of convertible arbitrage hedge funds, the dominant investors in the CB market since 2005 (see De Jong et al., 2013; Grundy and Verwijmeren, 2018). The presence of call risk exposes these investors to unsuccessful convertible arbitrages.

Whereas CB holders are prone to cluster their conversions when r is high, the conversions tend to be spread out over time when the underlying share prices are more volatile or when more cash dividends are paid to existing share holders as in **Figures 6**. Though the increments of σ increase the continuation values of conversion options and thus appreciate CB values (see King and Mauer, 2000, 2014), the presence of cash dividend payments (i.e., $\delta > 0$) would on the other hand deprive the continuation values to force early conversion (see Ingersoll, 1977; Asquith and Mullins, 1991). However, compared with conversion all at once early, conversion bit by bit is obviously a better strategy to preserve CB value, because it avoids too much share dilution that reduces the value of other unconverted CBs. Therefore, with the presence of dividends to the underlying shares as the CBs are not dividend-protected, allowing for sequential conversion policy indeed makes pronounced evaluation impact when the share prices are volatile. That again makes our evaluation framework



(a) (b)

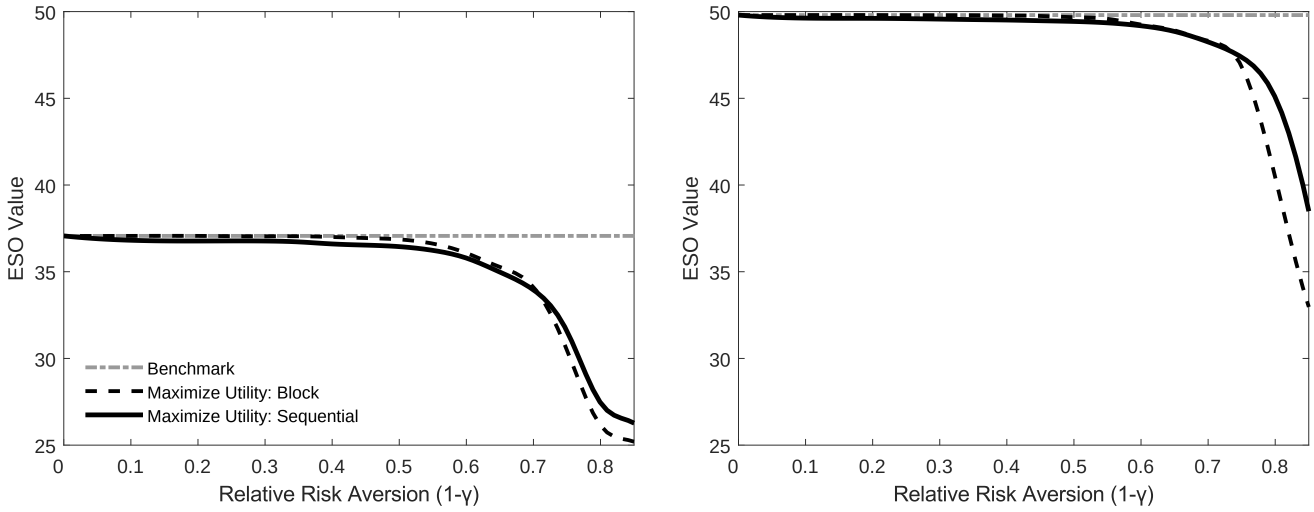
Figure 6: The CB Values under Different Asset Value Volatility and Payout Rates. All settings in panels (a) and (b) are the same as the benchmark scenario (denoted by node A) in **Figure 5** other than the asset value volatility and payout rate, respectively.

important, because empirical evidence shows that CBs are more likely to be issued by either unrated or speculative-grade firms (see Huang and Ramirez, 2010), whose share prices tend to be volatile.

4.2 The Impacts of Sequential Exercise Policy on ESO Values

Figures 7(a) illustrates the variation in ESO values for different choices of a representative employee's relative risk aversion when her β in **Equation (3)** is 0. Specifically, the employee has linear utility of each exercise payoff when she is risk-neutral (i.e., $1-\gamma=0$). In such scenario, utility-based and risk-neutral valuation of ESOs lead to same results, because the policy to maximize the expected utility of the intertemporal exercise payoffs is equivalent to the one to maximize the present value of the ESOs. Therefore, the three curves converge at $1-\gamma=0$ no matter how high the asset value volatility is (see the three overlapped curves in **Figures 8(a)**). The benchmark ESO value (the dot dashed horizontal lines) is implied by the no early exercise policy, because the issuing firm does not pay cash dividends to existing share holders. However, the two black curves diverge from the benchmark ESO value with the increments of relative risk aversion. This reveals the valuation impact of adopting the early exercise policy when the employee is risk-averse. Such pattern keeps existing as in **Figure 7(b)** even when the ESOs have sufficiently long life. Indeed, an employee's risk preferences may reduce the ESO values (see Hall and Murphy, 2002).

Though ESOs tend to be early exercised by a risk-averse employee, empirical evidence shows that the exercises are actually spread out over time (see Hemmer et al., 1994; Aboody, 1996; Huddart and Lang, 1996). Consistent with Jain and Subramanian (2004), **Figures 7** and **8(b)** display that, compared with the ESOs evaluated under sequential exercise policy (black solid curves), those evaluated under block one (black dashed curves) are underestimated when an employee's relative risk aversion exceeds a certain level or when the asset value volatility exceed a threshold. When the β turns into a positive number, the cash flows earned in the near future becomes preferable to those earned in the



(a) (b)

Figure 7: `ESO` **Values under Different Levels of Relative Risk Aversion.** In panels (a) and (b), the horizontal dot dashed gray lines indicate the benchmark `ESO` value, which is generated by the risk-neutral valuation and the representative `ESO` holder is only allowed for the block exercise policy. The black curves indicate the results generated by the utility-based valuation. In particular, the dashed curves illustrate the scenario that the `ESO` holder is only allowed for the block exercise policy. The solid curves illustrate the scenario that the sequential exercise policy is allowed. The exercise prices K_c of the `ESO`s are set to make the issuer's initial share price $S(0, V_0)$ identical to the K_c in the benchmark scenario; $r = 0.06$; $\eta = 0.08$; $\beta = 0$; the maturities T of the `ESO`s in panels (a) and (b) are 5 and 10 years, respectively; the number of outstanding `ESO`s N_c is set to 25. All other settings are identical to those in **Figure 5(a)**

distant future. That thus strengthens the incentives to adopt the early exercise policy and exaggerate the underestimation further. It can be observed in **Figure 8(c)** that the two black curves are slightly below the dot dashed lines to show the impact of positive β on `ESO` values. Greater relative risk aversion in **Figure 8(d)** further enhances early exercise incentives and thus enlarges the impact of β to make dashed curves well below the black solid ones for all chosen asset value volatility.

In sharp contrast with Jain and Subramanian (2004), our framework carefully considers capital injection and equity dilution when `ESO`s are exercised bit by bit. This allows us to approximate the `ESO` values capturing the employee's policy to trade her benefits from capital injection off against share dilution effect. Such tradeoff can makes significant valuation impact on `ESO`s as illustrated in **Figure 9**. It can be observed that the solid and dashed curves are getting divergent with the increments of the `ESO` outstanding amounts. In particular, the solid curve increases concavely to reflect that the dilution effect is salient enough to significantly suppress the underlying stock prices and thus the `ESO` values. Therefore, our framework may especially contribute to the valuation of `ESO`s when the issuing firm's capital structure contains substantial `ESO`s, such as the firms in high-technology industries (see Huddart and Lang, 1996).

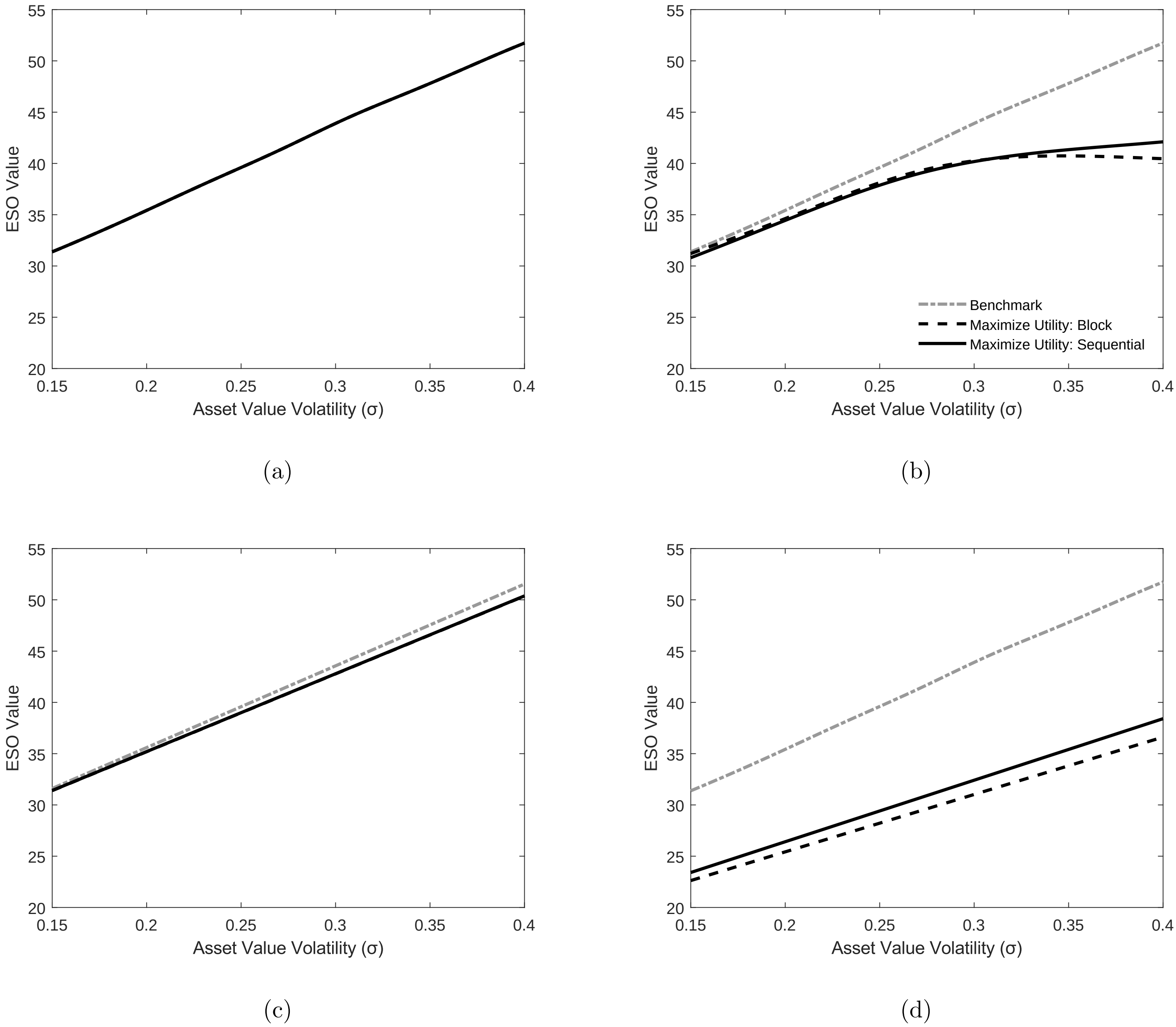


Figure 8: ESO Values under Different Asset Value Volatility. In panels (a) and (b), β is 0, and $1-\gamma$ is set to 0 and 0.6, respectively. Panels (c) and (d) illustrate the otherwise identical scenarios but β is set to 0.125. The dot dashed gray curves in all panels indicate the benchmark ESO values for different choices of asset value volatility. All other settings are identical to those in **Figure 7(a)**

5 Conclusion

This paper presents a novel framework, the "state-transition tree", to evaluate corporate securities when the granted options for the security holders are sequentially exercised. In particular, the tree is developed to capture the entire paths of intertemporal exercises due to share dilution and (or) capital injection that changes the values of underlying assets when options are exercised. The structural credit risk models can be implemented by this framework, and it can be well applied to a risk-neutral valuation of a CB as well as to a utility-based valuation of ESOs for which close-form solutions are unavailable. It then shows salient valuation impact when its pricing results are compared to the ones

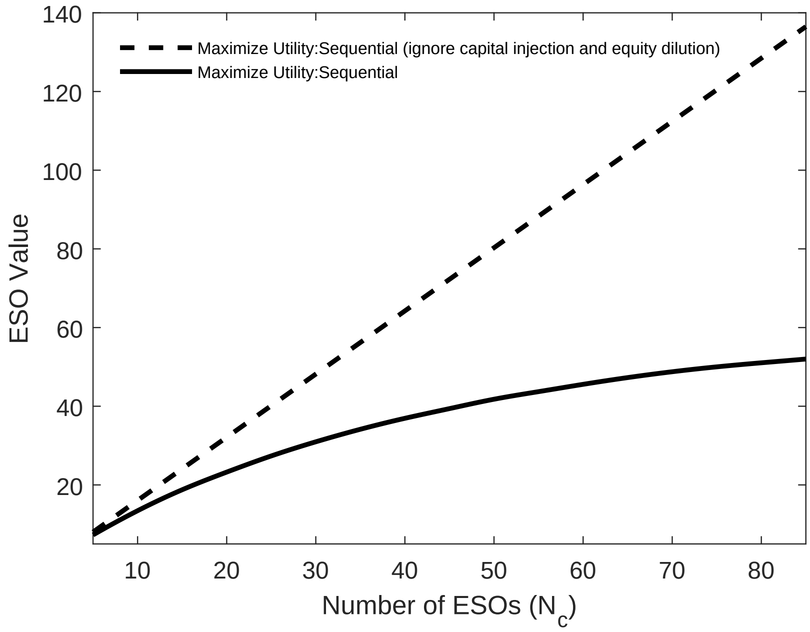


Figure 9: ESO **Values with or without Capital Injection and Equity Dilution.** The solid curves are the results considering capital injection and equity dilution once ESOs are exercised, whereas the dashed curves represent the opposite scenarios. All settings are identical to those in **Figure 8(d)** other than the number of outstanding ESOs.

from the traditional models with the block exercise constraint.

References

Aboody, D. (1996). Market valuation of employee stock options. *Journal of accounting and economics 22*(1-3), 357–391.

Acharya, v. and J. Carpenter (2002). Corporate bond valuation and hedging with stochastic interest rates and endogenous bankruptcy. *Review of Financial Studies 15*(5), 1355–1383.

Asquith, P. and D. W. Mullins (1991). Convertible debt: Corporate call policy and voluntary conversion. *The Journal of Finance 46*(4), 1273–1289.

Attaoui, S. and P. Poncet (2013). Capital structure and debt priority. *Financial Management 42*(4), 737–775.

Bechmann, K. L., A. Lunde, and A. A. Zebedee (2014). In-and out-of-the-money convertible bond calls: Signaling or price pressure? *Journal of Corporate Finance 24*, 135–148.

Bhattarcharya, M. (2012). Convertible securities: their structures, valuation, and trading. *In: Fabbozzi, Frank J.(Ed.), The Handbook of Fixed Income Securities, 8th ed.*, 899–945.

Black, F. and J. Cox (1976). Valuing corporate securities: Some effects of bond indenture provisions. *The Journal of Finance 31*(2), 351–367.

Bris, A., I. Welch, and N. Zhu (2006). The costs of bankruptcy: Chapter 7 liquidation versus chapter 11 reorganization. *Journal of Finance 61*, 1253–1303.

Briys, E. and F. De Varenne (1997). Valuing risky fixed rate debt: An extension. *Journal of Financial and Quantitative Analysis 32*(2), 239–248.

Broadie, M. and Ö. Kaya (2007). A binomial lattice method for pricing corporate debt and modeling chapter 11 proceedings. *Journal of Financial and Quantitative Analysis 42*(2), 279–312.

Bühler, W. and C. Koziol (2002). Valuation of convertible bonds with sequential conversion. *Schmalenbach Business Review 54*(4), 302–333.

Carpenter, J. N., R. Stanton, and N. Wallace (2010). Optimal exercise of executive stock options and implications for firm cost. *Journal of Financial Economics 98*(2), 315–337.

Chen, H. (2010). Macroeconomic conditions and the puzzles of credit spreads and capital structure. *The Journal of Finance 65*(6), 2171–2212.

Chen, L., Z. Da, and R. Priestley (2012). Dividend smoothing and predictability. *Management science 58*(10), 1834–1853.

Chen, N., M. Dai, and X. Wan (2013). A nonzero-sum game approach to convertible bonds: Tax benefit, bankruptcy cost, and early/late calls. *Mathematical Finance 23*(1), 57–93.

Collin-Dufresne, P. and R. S. Goldstein (2001). Do credit spreads reflect stationary leverage ratios? *Journal of Finance 56*(5), 1929–1957.

Constantinides, G. M. and B. D. Grundy (1987). Call and conversion of convertible corporate bonds: theory and evidence. *Working Paper*.

Cox, J. C., S. A. Ross, and M. Rubinstein (1979). Option pricing: A simplified approach. *Journal of Financial Economics 7*(3), 229–263.

Dai, T. and Y. Lyuu (2010). The bino-trinomial tree: A simple model for efficient and accurate option pricing. *The Journal of Derivatives 17*(4), 7–24.

De Jong, A., E. Duca, and M. Dutordoir (2013). Do convertible bond issuers cater to investor demand? *Financial Management 42*(1), 41–78.

Duffie, D. (1996). *Dynamic asset pricing theory.* Princeton University Press.

Emanuel, D. C. (1983). Warrant valuation and exercise strategy. *Journal of Financial Economics 12*(2), 211–235.

Fan, H. and S. M. Sundaresan (2000). Debt valuation, renegotiation, and optimal dividend policy. *Review of financial studies 13*(4), 1057–1099.

Figlewski, S. and B. Gao (1999). The adaptive mesh model: a new approach to efficient option pricing. *Journal of Financial Economics 53*(3), 313–351.

Geske, R. (1977). The valuation of corporate liabilities as compound options. *Journal of Financial and Quantitative Analysis 12*(04), 541–552.

Grundy, B. D. and P. Verwijmeren (2018). The buyers' perspective on security design: Hedge funds and convertible bond call provisions. *Journal of Financial Economics 127*(1), 77–93.

Hall, B. J. and K. J. Murphy (2002). Stock options for undiversified executives. *Journal of accounting and economics 33*(1), 3–42.

Harrison, J. M. and D. M. Kreps (1979). Martingales and arbitrage in multiperiod securities markets. *Journal of Economic theory 20*(3), 381–408.

He, Z. and W. Xiong (2012). Rollover risk and credit risk. *Journal of Finance 67*(2), 391–430.

Hemmer, T., S. Matsunaga, and T. Shevlin (1994). Estimating the'fair value'of employee stock options with e. *Accounting Horizons 8*(4), 23.

Huang, R. and G. G. Ramirez (2010). Speed of issuance, lender specialization, and the rise of the 144a debt market. *Financial Management 39*(2), 643–673.

Huddart, S. and M. Lang (1996). Employee stock option exercises an empirical analysis. *Journal of Accounting and Economics 21*(1), 5–43.

Ingersoll, J. E. (1977). An examination of corporate call policies on convertible securities. *The Journal of Finance 32*(2), 463–478.

Jain, A. and A. Subramanian (2004). The intertemporal exercise and valuation of employee options. *The Accounting Review 79*(3), 705–743.

Ju, N. and H. Ou-Yang (2006). Capital structure, debt maturity, and stochastic interest rates. *The Journal of Business 79*(5), 2469–2502.

Kim, D. H. and D. Stock (2014). The effect of interest rate volatility and equity volatility on corporate bond yield spreads: A comparison of noncallables and callables. *Journal of Corporate Finance 26*, 20–35.

Kim, I., K. Ramaswamy, and S. Sundaresan (1993). Does default risk in coupons affect the valuation of corporate bonds?: A contingent claims model. *Financial Management*, 117–131.

King, T.-H. D. and D. C. Mauer (2000). Corporate call policy for nonconvertible bonds*. *The Journal of Business 73*(3), 403–444.

King, T.-H. D. and D. C. Mauer (2014). Determinants of corporate call policy for convertible bonds. *Journal of Corporate Finance 24*, 112–134.

Kuehn, L.-A. and L. Schmid (2014). Investment-based corporate bond pricing. *Journal of Finance, forthcoming*.

Lando, D. (2004). *Credit risk modeling: theory and applications.* Princeton University Press.

Leary, M. T. and R. Michaely (2011). Determinants of dividend smoothing: Empirical evidence. *The Review of Financial Studies 24*(10), 3197–3249.

Leland, H. E. (1994). Corporate debt value, bond covenants, and optimal capital structure. *The journal of finance 49*(4), 1213–1252.

Leland, H. E. (2004). Predictions of default probabilities in structural models of debt. *Working paper*.

Liu, L.-C., T.-S. Dai, and C.-J. Wang (2016). Evaluating corporate bonds and analyzing claim holders' decisions with complex debt structure. *Journal of Banking & Finance 72*, 151–174.

Longstaff, F. A. and B. A. Tuckman (1994). Calling nonconvertible debt and the problem of related wealth transfer effects. *Financial Management 23*(4), 21–27.

Merton, R. C. (1974). On the pricing of corporate debt: The risk structure of interest rates. *The Journal of finance 29*(2), 449–470.

Modigliani, F. and M. Miller (1958). The cost of capital, corporation finance and the theory of investment. *The American economic review 48*(3), 261–297.

Rogers, L. and J. Scheinkman (2007). Optimal exercise of executive stock options. *Finance and Stochastics 11*(3), 357–372.

Spatt, C. S. and F. P. Sterbenz (1988). Warrant exercise, dividends, and reinvestment policy. *The Journal of Finance 43*(2), 493–506.

Wang, C.-J., T.-S. Dai, and Y.-D. Lyuu (2014). Evaluating corporate bonds with complicated liability structures and bond provisions. *European Journal of Operational Research 237*(2), 749–757.

Appendix A Robustness Checks

Before investigating the impact of allowing for intertemporal exercise policy on CB and ESO values, it is critical to verify that the evaluation framework produces accurate and stable pricing results. Although pricing results generated via tree methods are expected to converge to theoretical values with the increment in the number of the time steps of the tree and forest (see Duffie, 1996), improper tree structure adjustment can reduce accuracy. Prior literature such as Broadie and Kaya (2007) and Wang et al. (2014) examine the robustness of their methods by exhibiting that their pricing results converge to those produced by analytical formulas. However, analytical formulas are not available for corporate securities allowing both for block and intertemporal exercises of the granted options when the corresponding share dilution and capital injection are considered.

Rather than confirming the correctness of each pricing results directly, we follow Liu et al. (2016) to indirectly check the rationality of the pricing results as a whole by exploiting the capital irrelevance theory proposed by Modigliani and Miller (1958). That is, in a perfect and frictionless market, the market value of a firm is independent of the capital structure. Our evaluation framework should generate the same market value for the firm despite the changes of its capital structure under otherwise identical conditions. Indeed, given the capital market is frictionless (i.e., no taxes and bankruptcy cost), our numerical results in Tables 1 to 4 exhibit that the levered firm value, which is equal to the lump sum values of all contingent claim s generated by our framework, is equal to the initial unlevered firm value V_0 (400 in this experiment) under the different scenario listed in the first row.

Scenario	Callable CB, Block			Callable CB, Intertemporal			Noncallable CB, Block			Noncallable CB, Intertemporal		
Time steps	**4**	**12**	**36**	**4**	**12**	**36**	**4**	**12**	**36**	**4**	**12**	**36**
SB	106.241	106.649	106.790	106.240	106.649	106.776	106.266	106.677	106.814	106.265	106.677	106.802
CB	100.311	100.000	99.733	100.322	100.003	99.740	108.810	108.724	108.763	108.816	108.727	108.770
E	193.448	193.351	193.477	193.438	193.348	193.484	184.924	184.599	184.433	184.919	184.596	184.428

Table 1: Robustness check for CB valuation. The bond prices and the corresponding equity values (in the last row) generated under the state-transition tree are examined under four different scenarios denoted by the first row: the valuation of a callable CB when a block or a intertemporal exercise policy is allowed and the otherwise identical valuation of a noncallable CB. Time steps in the second row denotes the number of time steps used to partition the one-year time span of the state-transition tree. Two outstanding $100 bonds, SB and CB, are assumed to be issued by the same hypothetical issuer with bond maturity $T = 10$ years and coupon rates of 0.09 and 0.07, respectively. The current issuing firm's asset value V_0 is 400, its volatility σ is 0.22, the initial volume of outstanding shares $N = 50$, the risk-free rate r is 0.08, and the δ in the payout policy **(P.1)** is 0.06. If the CB is callable, the call price K_c of the CB is set to its 100, and the call protection period ν is set to 1 year. To confirm the correctness of our evaluation framework, the tax rate τ and the bankruptcy cost ω are set to 0. The minimum increment of the exercise volume when a intertemporal exercise policy is allowed is set to 20%. The conversion ratio is set to make the callable CB be priced at par (colored in red) when a intertempoal exercise policy is not allowed and when the Time steps is equal to 12.

The first two columns in Table 1 indicate the scenarios that the CB is callable and the intertemporal conversion policy is only allowed for the latter scenario. The other two columns refer to the otherwise identical scenarios but the CB is noncallable. It can be observed that the lump sum of the SB and CB values plus the corresponding equity value listed in each column is about 400 regardless of the Time steps of the trees, the presence of call provisions, and the allowance of intertemporal exercises. Notice that, *ceteris paribus* under this premise, the values of callable CBs are less than that of otherwise identical noncallable CBs due to the presence of call provisions that favors equity holders.

Furthermore, CBs with the consideration of intertemporal conversions have slightly greater values, for the evaluation framework allows the CB holders to minimize the dilution effect to increase the values of other unconverted CBs. If the evaluation framework allows the CB holders to control the dilution effect in more sophisticated way by reducing the minimum increment of the conversion volume, the CB values will increase even further as in Table 2 under all otherwise identical scenarios.

A. Valuation of a Callable CB

Scenario	**100%** (Block)	**50%**	**20%**	**10%**	**5%**	**1%**
SB	106.649	106.649	106.649	106.649	106.611	106.593
Callable CB	100.000	100.002	100.003	100.003	100.012	100.018
E	193.351	193.349	193.348	193.348	193.377	193.389

B. Valuation of a Noncallable CB

Scenario	**100%** (Block)	**50%**	**20%**	**10%**	**5%**	**1%**
SB	106.677	106.677	106.677	106.677	106.662	106.619
Noncallable CB	108.724	108.726	108.727	108.727	108.729	108.741
E	184.599	184.597	184.596	184.596	184.609	184.640

Table 2: Robustness checks when the minimum increment of the exercise volume is different. The number of time steps used to partition the one-year time span of the state-transition tree is set to 12. All settings are the same as those in **Table 1** other than the minimum increment of the exercise volume. In particular, 100% refers to the scenario that the evaluation framework only allows for block conversion policy. Others indicate that the framework allows for intertemporal conversion policy with the minimum increment of the conversion volume equal to 50%, 20%,..., and 1% of total CB face value.

Scenario	Risk-neutral ($\beta = 0$, $\gamma = 1$)			Block ($\beta = 0.125$, $\gamma = 0.4$)			Intertemporal ($\beta = 0.125$, $\gamma = 0.4$)		
Time steps	**4**	**12**	**36**	**4**	**12**	**36**	**4**	**12**	**36**
SB	205.398	206.520	206.897	206.487	207.579	208.031	206.295	207.481	207.887
ESO	49.857	49.497	49.385	28.932	28.483	27.749	31.848	29.848	29.002
E	144.745	143.983	143.718	164.581	163.938	164.220	161.857	162.671	163.111

Table 3: Robustness checks for ESO valuation. The bond prices, ESO values, and the corresponding equity values generated under the state-transition tree are examined under three different scenarios denoted by the first row: the risk-neutral valuation of ESOs and the utility-based valuation of ESOs when a block or a intertemporal exercise policy is allowed. Time steps in the second row denotes the number of time steps used to partition the one-year time span of the state-transition tree. The expected rate of return of the issuing firm's asset value η in the real world probability measure is 0.08. The number of outstanding ESOs N_c is set to 25. The exercise price K_c of the ESOs is set to make the issuer's initial share price identical to the K_c when the risk-neutral valuation is applied and when the Time steps is equal to 12. The payout policy is **(P.2)**, and we suppose that the ESO issuing firm does not pay dividends. All other settings are the same as those in **Table 1**.

On the other hand, the first column of Table 3 indicates the risk-neutral valuation of ESOs, and the other two refer to the utility-based valuation but the intertemporal exercise policy is only allowed in the latter scenario. It can also be observed that the lump sum of the SB and ESO values plus the corresponding equity value listed in each column is about 400 regardless of the Time steps of the forests, utilization of risk-neutral or utility-based valuation, and the allowance of intertemporal exercises. In particular, *ceteris paribus* under this premise, the difference in ESO values between the first two columns reveal the impact of risk aversion on the ESO exercise timings and thus the ESO values. Lower ESO values imply earlier exercise timings that maximize risk-averse ESO holders' utility rather than ESO values. In addition, the difference in ESO values between the second and third columns reveal the pure impact of allowing for intertemporal exercise policy. Once the exercises of ESOs are spread out over time, the dilution effect is alleviated to increase the values of other unexercised ESOs.

A. Risk-neutral valuation of ESOs ($\beta = 0\,, \gamma = 1$)

Scenario	**100%** (Block)	**50%**	**20%**	**10%**	**5%**	**1%**
SB	206.520	206.520	206.520	206.520	206.520	206.520
ESO	49.497	49.497	49.497	49.497	49.497	49.497
E	143.983	143.983	143.983	143.983	143.983	143.983

B. Utility-based valuation of ESOs ($\beta = 0.125\,, \gamma = 0.4$)

Scenario	**100%** (Block)	**50%**	**20%**	**10%**	**5%**	**1%**
SB	207.579	207.546	207.481	207.389	207.269	206.876
ESO	28.483	28.897	29.848	31.009	32.784	40.702
E	163.938	163.557	162.671	161.602	159.947	152.422

Table 4: Robustness checks when the minimum increment of the exercise volume is different. The number of time steps used to partition the one-year time span of the state-transition tree is set to 12. All settings are the same as those in **Table 3** other than the minimum increment of the exercise volume. In particular, 100% refers to the scenario that the evaluation framework only allows for block exercise policy. Others indicate that the framework allows for intertemporal exercise policy with the minimum increment of the exercise volume equal to 50%, 20%,..., and 1% of total outstanding ESOs.

If the evaluation framework allows the representative ESO holders to adopt a more sophisticated exercise policy by reducing the minimum increment of the exercise volume as in Table 4, the ESO values can increase even further under otherwise identical scenarios. When the risk-neutral valuation of ESOs is applied, the ESO values generated by our framework are independent of the settings of the minimum increment of the ESO exercise volume as exhibited in Table 4A. This implies that the optimal policy for a risk-neutral ESO holder is to exercise all of the ESOs at a single date, and that date is the ESO maturity date if the issuing firm does not pay dividends to the underlying stocks (see Jain and Subramanian, 2004; Carpenter et al., 2010). However, when the utility-based valuation of ESOs is applied, the ESO value will increase as exhibited in Table 4B if a risk-averse ESO holder is allowed to spread the exercise of her ESOs out over time in a more sophisticated way.